Industrialization, poverty traps, and the Dutch disease: a dual model

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The paper develops a Ricardo-Viner-Jones model of a small open economy, in which agriculture is subject to diminishing returns and market-clearing wages, while increasing returns and efficiency wages prevail in industry. The asymmetric interaction of the two sectors is such that the model displays multiple equilibria and a low-development trap under plausible parametrization. Additionally, comparative statics shows how a Dutch disease emerges, in which parametric increases of agricultural TFP broaden the basin of attraction of the low-equilibrium and decrease the steady state level of capital stock (and wages) for the stable equilibrium of full industrialization. On the contrary, increases of industrial TFP reduce the basin of attraction of the low-equilibrium and increase the steady state level of capital stock (and wages) for the stable equilibrium of full industrialization.

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JEL codes: F43, O11, O14, O41.

I. *Introduction*

In the corpus of literature belonging to the so-called "high development theory"[1] industrialization and structural change have always played a crucial role. Along the lines of Smith or Ricardo, the authors of the Fifties have unanimously acknowledged "agriculture industry shift"[2] as one of the main elements (if not the main element) of the development process, be it in view of the different scopes for productivity growth in the primary sector vis-à-vis manufacturing, or because of the more

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[1]This label has been used by Krugman (1992) to refer to those works of the Fifties and early Sixties, that analyzed the economies of underdeveloped nations and their challenges to modernization. Among the most famous contributors to this line of analysis one can cite Rosenstein-Rodan, Nurkse, Lewis, Leibenstein, Myrdal, Hirschmann, Ranis and Fei. Byres (2003) grouped these same authors under the name of "classical development economists", emphasizing the strong intellectual linkages between these authors and the classics such as Smith, Malthus and Ricardo.

[2]The notion refers to the well-known decline of agriculture in terms of both contribution to GDP and employment share, as income per capita increases and nations grow richer. For some empirical evidence at this regard see among others Kuznets (1966), Chenery and Syrquin (1975), Syrquin (1989), Taylor (1989) and Bhaduri (1993 and 2003).

efficient institutional arrangements intrinsic to the "capitalistic sector" as opposed to the "traditional one". In this developmental perspective, four elements characterize the "high development theory": (i) the role of dualism in underdeveloped economies, (ii) the importance of labor supply elasticity, (iii) the emphasis on sectoral balances and on the economic interactions among sectors, and (iv) the idea that backwardness may be a state of equilibrium.

As for dualism, in the "high development theory" organizational asymmetries between agriculture and industry are assumed to operate so that the modern sector faces an elastic labor supply and consequently can expand at relatively favorable conditions[3]. Further, Rosenstein-Rodan (1943), Nurkse (1953), Scitovsky (1954) and others follow the early insights of Young (1928)[4] in pointing out the importance of "technological asymmetries" across sectors, with special reference to the presence of increasing returns in industry. Both these factors imply complex interactions between the traditional rural-based agriculture and the modern city-based industry, with the former acting as a bargaining sector releasing resources (in terms workers and wealth) for the expansion of the latter, but also supplying to urban centers key necessary goods (post-keynesians would say wage goods). Coming to the last element, as argued among others by Rosenstein-Rodan (1943), Nurkse (1953) and Leibenstein (1957), the presence of increasing returns and of elastic inputs' supply makes a strong case for the existence of multiple equilibria and possibly of low-development traps[5].

Be it for the lack of clear successes in terms of policy decisions to be ascribed to the "classical development economics", or for the theoretical difficulty to reconcile increasing returns with competitive market structures, the Neoclassical approach[6] becomes dominant during he Sixties, and rather overlooks the four above aspects. The predominant use of aggregate models dismisses by definition the role of structural change and sectoral balances, to focus on reproducible factors' accumulation, and on the determinants of the steady state[7]. Compared to the "high development theory", the aggregate growth approach entails also a different role for labor supply elasticity, which basically becomes tantamount to growth in the labor force rather than being the outcome of sectoral interactions. Finally, notwithstanding several important contributions on the role of increasing returns and learning by doing, during the Sixties the mainstream approach to growth becomes that of the convex economy

[3]The epitome of this reasoning is surely Lewis's well-known model of unlimited labor supply, but similar notions can be found in Nurkse (1953), Leibenstein (1957) and to some extent even in Rosenstein-Rodan (1943). Note that the concept of disguised unemployment justifies in the "high development theory" what the classical reasoning achieves through malthusian law of population (or Marx's "reserve army"): the elastic nature of the labor supply for the industrial sector.

[4]Young himself refers to Smith's reknown notion of the division of labor to justify the presence of increasing returns in the industrial sector as opposed to decreasing or constant returns to scale in the agriculture. However, in Young the emphasis moves from the economies of specialization and the reduction of downtimes of the first three chapters of the Wealth of Nations, to the fixed costs of increasing return technologies prevailing in the "capitalistic or roundabout methods of production".

[5]Note that while the notions of poverty traps and multiple equilibria emerged explicitly in the early Fifties, they can actually be traced back - though in an implicit form - even to classical authors such as Adam Smith and Thomas Malthus. In the "Early draft of part of the Wealth of Nations" (1763) page 579 the Smith argues: "That is easier for a nation, in the same manner as for an individual, to raise itself from a moderate degree of wealth to the highest opulence, than to acquire this moderate degree of wealth.". As for reverend Thomas Malthus, he observes at page 310 of his Principles of political economy: "...that there are many countries, not essentially different..., which yet, with nearly equal natural capabilities, make very different progress in wealth."

[6]As attains growth theory, the epitome of the neoclassical approach is the Solow-Swan model (Solow (1956), Swan (1956)), later re-elaborated by Cass (1965) and Koopmans (1965), who endogenize saving decisions into an intertemporal optimization framework.

[7]Such flaw could indeed be misleading when analyzing economies that are indeed undergoing a process of industrialization rather than of "homothetic growth". Not surprisingly, the empirical literature has found growing evidence of the limited explanatory power of the so-called "augmented Solow regressions" in the case of poor countries, and has suggested the need to go beyond the common linear specification of the growth process commonly used in cross-country "Barro regressions". See Durlauf and Johnson (1995); Durlauf, Kourtellos and Minkin (2001) and Durlauf, Johnson and Temple (2005).

converging to a stable and unique steady state.

While in the Seventies the economy of information highlights the sensitivity of Neoclassical results to strong informational assumptions, it is basically the advent of the endogenous growth theory, with its emphasis on the role of knowledge and human capital, that brings back to the center of the attention the notion of increasing returns along with their implications for multiple equilibria and poverty traps, leading to what has been called a "counter-counterrevolution in development theory"[8]. The renewed interest in poverty traps comes also under the pressure of the empirical literature, which increasingly questions the validity of the conditional convergence hypothesis, in favor of more complex dynamics able to generate convergence clubs and twin peaked distributions. Cross-country regressions have for long confirmed that economies tend to converge to their own steady state at a rate consistent with the "augmented Solow model", once controlling for the determinants of the steady state itself: typically the saving rate, the initial level of human capital, political stability and degree of price distortion[9]. Despite this, several econometric works accounting for parameters heterogeneity across countries (rather than relying on a common linear specification, as in standard growth regressions) find evidence of *multiple growth regimes and convergence clubs* formation[10]. On the other hand, the existence of convergence clubs seems confirmed also by non-parametric inference about the cross-country distribution of GDP per capita, and on the "distribution dynamics" of Markovian growth processes[11]. While not necessarily incompatible with neoclassical growth models, the existence of convergence clubs comes rather at odds with the traditional neoclassical paradigm à la Solow or with its "augmented version", while it rationalizes immediately the observed absolute σ-divergence across countries.

In light of the long standing debate summarized above, in this paper we aim at building a theoretical model able to reconcile the neoclassical theory of growth, some insights of the endogenous growth literature, and the industrialization perspective derived from the "high development theory". We do so by developing a specific-factor macro model à la Ricardo-Viner-Jones, which displays multiple equilibria and poverty trap under plausible parametrization. We prove how the explicit consideration of labor supply elasticity in a two-sector set-up is crucial for the existence of multiple equilibria even in an open economy setting with two tradeable goods[12], and additionally show how the specialization of the economy is structurally linked to the multiplicity of steady states.

The paper is organized as follows: section II outlines the macro model and the determination of the equilibria, section III explains the effect of exogenous technical progress (here intended as a parametric increase of sectoral TFP) and highlights the working of a Dutch disease mechanism, section IV concludes.

II. *The model*

PREFERENCES

Consider a small open economy, consisting of two tradeable sectors, agriculture and industry, that produce respectively a consumption good (call it food) and manufactures, which can be alternatively consumed or invested. Consumers' preferences across goods are described by a Stone-Geary utility

[8]See Krugman (1992).

[9]See among others Barro (1991); Mankiw, Romer, Weil (1992); Barro, Sala-i-Martin (1995); Sala-i-Martin (1996); Easterly (2006).

[10]See Durlauf and Johnson (1995); Durlauf, Kourtellos and Minkin (2001) and Durlauf, Johnson and Temple (2005).

[11]See Bianchi (1997); Quah (1993 and 1996); Desdoigt (1999); Azariadis, Stachurski (2005); Azariadis (2005).

[12]Results similar to those presented here had been previously obtained in the case of open economies that included also a non-tradeable intermediate goods' sector, operating under a non-convex technology. See Ros and Skott (1997), Hoff (2000) and Ros (2000).

function:

$$U = (X_a^c - Z)^{\alpha} \ (X_i^c)^{1-\alpha};$$

where X_a^c and X_i^c are respectively the amount of food and manufactures consumed, Z is the minimum required amount of food[13], while α represents the food expenditure share applied to the supernumerary income (the income remaining after the purchase of the minimum quantity of food Z). Through standard utility maximization under budget constraint, representative consumers' demand can be shown to be:

$$\frac{\alpha}{1-\alpha} \ \frac{X_i^c}{X_a^c - Z} = P_a^*; \qquad (1)$$

where prices are set at the international level, since the small country under question operates in the world market as price-taker, and where P_i^* has been normalized to 1 by an appropriate choice of unit (so P_a^* represents the agricultural terms of trade). Consistently with the above specification of preferences, the price index $\underline{P}^*$ is defined as $\underline{P}^* \equiv P_a^* Z + \psi \ (P_a^*)^{\alpha}$, where the constant ψ is equal to $\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$.

TECHNOLOGIES

In line with the literature on dual economies, we employ a Ricardo-Viner-Jones approach to describe technologies, so as to emphasize the technological asymmetries between agriculture and industry. We assume the agricultural sector to operate through a backward technology that uses labor and land, but, unlike manufacturing, it has no scope for reproducible inputs[14]. The food production function is hence given by

$$X_a^s = A_a L_a^{1-b}; \qquad 0 \leq b < 1 \qquad (2)$$

where X_a^s denotes food output, L_a the labor employed in agriculture, $(1-b)$ and A_a are technological parameters describing respectively the degree of returns to labor and the sectoral TFP (which in the case of agriculture summarizes both technological factors but also geographical and climatic conditions). The restriction on b derives from the hypothesis that land endowment is fixed even in the long-run[15], and implies decreasing returns to labor ($b = 0$ is a limiting case, representing constant return to labor).

The modern industrial sector, instead, utilizes labor (in efficiency units) and capital in the production of manufactures. Furthermore, the manufacturing sector is assumed to exhibit increasing returns to scale due to *Marshallian external economies* associated with capital stock, and captured by a *Kaldor-Verdoorn coefficient*, which rationalizes the positive externality stemming from "capital-embodied-knowledge". In other words, we take the stock of knowledge as proxied by the average economy-wide stock of capital, and postulate that capital accumulation translates automatically into improvements of the knowledge base and hence of the industrial TFP at the constant rate μ (precisely the Kaldor-Verdoorn coefficient). This formalization is equivalent to a learning by doing process, in which the cumulative gross investment represents the index of experience, and knowledge depreciates at the same rate as physical capital[16].

[13]In order for the Stone-Geary utility function to be meaningful, it should be assumed that $X_a^c > Z$ over the whole domain, meaning that in all cases food consumption exceeds the minimum required amount Z (this hypothesis will be important at a later stage to determine the sign of several magnitudes). Clearly, by setting $Z = 0$ one falls back in the homotetic preference case, and the utility function turns into a standard Cobb-Douglas.

[14]Such a technology is evidently unappropriate for high and middle income countries displaying capital-intensive techniques of cultivation, however it represents a suitable approximation for less developed countries (LDC). This widely adopted assumption, however restricts the relevance of the present model to those countries, where subsistence agriculture is especially widespread and the scarce physical capital is employed in non-agricultural activities: above all South Asian and Sub-Saharan African countries.

[15]The fixed argument "land" has been omitted from the production function to lean down the notation.

[16]In this respect, the present approach to learning-by-doing differs from both Arrow's original view (1962), in which experience is also proxied by cumulative gross investment but without knowledge depreciation, as

In accordance with the previous discussion, suppose that the industrial production function is described by

$$X_i^s = A_i \tilde{K}^{\mu} K^{\beta} \left(E_{(w_i,w_a)} L_i\right)^{1-\beta}; \qquad \mu > 0, \quad 0 < \beta < 1;$$

where X_i^s, L_i and K denote respectively manufacturing output, industrial labor and capital stock, the function $E_{(w_i,w_a)}$ represents labor efficiency, the parameters β, $(1-\beta)$ and A_i are respectively the capital and labor shares, and the industrial TFP, and finally $\tilde{K}^{\mu}$ represents the external positive effect of capital accumulation, $\tilde{K}$ being the average capital stock of our economy.

Since technological economies are *external* to each firm (knowledge is assumed to be non-rival and non-excludable, so that the experience acquired by one firm spills over *completely* and *immediately* to all the others[17]), one can argue that in equilibrium the average capital stock of the economy will match that of the representative firm. Accordingly, the industrial production function can be rewritten as

$$X_i^s = A_i K^{\mu+\beta} \left(E_{(w_i,w_a)} L_i\right)^{1-\beta}; \qquad \mu > 0, \quad 0 < \beta < 1. \tag{3}$$

Ours is a generalization of the standard AK technology: as long as $\mu > 0$ it displays aggregate increasing returns, though not necessarily constant or increasing returns to capital, as typically assumed in AK models or in other models of endogenous growth à la Romer[18].

Considering the technologies for agriculture and industry, it is straightforward to see that capital accumulation will not trigger a "homothetic growth" for the economy as a whole, precisely because in our dualistic set-up reproducible inputs are specific to only the modern industrial sector. Unlike in aggregate models, here capital accumulation affects asymmetrically the marginal productivity of labor in agriculture and manufacturing, leaving the burden of equilibrium adjustment to labor reallocation, changes in the wage levels, and capital-labor substitution (in industry)[19]. At the same time, resource reallocation across sectors determines a change in output composition and employment shares.

DISTRIBUTION AND LABOR MARKET

In addition to the technological asymmetries, distributive issues and "organizational asymmetries" between agriculture and industry play a key role in the present model, especially as concerns the labor market. Rather than following the debated hypothesis that wages in the traditional sector are determined à la Lewis by the average productivity of labor, we instead assume perfect competition among rentiers and laborers, so that the former hire all available workers and pay them at a wage rate equal to their marginal revenue product[20]. Analytically we will thus have:

$$W_a = (1-b)\, A_a \,(L_a)^{-b}\, P_a^*; \tag{4}$$

well as from recent models of structural change that disregard the idea of capital embodied knowledge and relate the learning process to cumulative output (for instance Krugman 1987, Stokey 1988, Matsuyama 1992).

[17]Despite the caveats about some more realistic refinements of the learning by doing process, the hypothesis of immediate and complete spillovers is widely used in the literature (see Krugman 1987, Matsuyama 1992, 2002, Stokey 1988) for it allows to concentrate on the impact of increasing returns without further analytical complications as regards the market structure.

[18]In this way, our generalization of AK models overcomes the problem of excessive sensitivity to restrictive parametrization, as increasing returns to capital arise here only if $\mu > 1-\beta$, with equality yielding constant returns to capital. See Stiglitz (1992) and Solow (1994) for a critique of AK models in this respect.

[19]Recall that adjustments in prices of the final goods have been ruled out by the "small country assumption", under which traded goods are priced domestically as in the international market.

[20]Maintaining the Lewisian assumption would not change qualitatively the conclusion of our model, but simply reduce the wage gap across sectors (since the average revenue product exceeds the marginal one in agriculture) and the scope for labor re-allocation towards industry, thus shortening the "dualistic" phase. One would however end up postulating the equivalence of rural wage and average labor productivity in agriculture even during the mature phase of the economy, or otherwise need to explain what triggers the change in the distributional rules at a certain point in time.

and

$$R = b\,A_a\ (L_a)^{1-b}\,P_a^* = \frac{b}{1-b}\,W_a\,L_a; \tag{5}$$

where W_a represents the rural wage in nominal terms and R the total rents.

Organizational dualism comes into play as regards wage determination in the industrial sector, where we assume the existence of an efficiency mechanism, linking labor productivity with the real wage received[21]. In light of such linkage, the problem faced by industrial entrepreneurs will be

$$\max_{L_i, Wi} [\Pi] = A_i\,K^{\mu+\beta}\left(E_{(w_i,w_a)}\ L_i\right)^{1-\beta} - L_i\,W_i; \quad \textit{subject to} \quad Wi \geq W_a$$

where upper-case W indicates wages in nominal terms (lower-case w are expressed in real terms), and $E_{(w_i,w_a)}$ is a non-decreasing function relating workers' efficiency with their real wage, and with the real wage they could get if working in agriculture. Notably, the problem faced by industrial entrepreneurs is a constrained maximization, since they cannot hire any worker at a wage lower than the reservation wage laborers could get in agriculture.

To specify the effort function, we follow Akerlof's interpretation of labor contracts as partial gift exchanges, in postulating that $E_{(w_i,w_a)}$ reflects those sociological considerations (including the real wages paid in the other sector of the economy) that govern the determination of work norms, and regulate labor productivity. Accordingly, we suppose that

$$E(W_i) = \begin{cases} 0; & \text{for } W_i < \omega^{\frac{1}{d}} W_a^{\gamma} \underline{P}^{*\,1-\gamma} \\ \left[\dfrac{W_i/\underline{P}^*}{(W_a/\underline{P}^*)^{\gamma}}\right]^d - \omega; & \text{for } W_i \geq \omega^{\frac{1}{d}} W_a^{\gamma} \underline{P}^{*\,1-\gamma} \quad 0 < d, \gamma < 1;\ \omega > 0; \end{cases} \tag{6}$$

in which the parameter ω implies a minimum threshold to obtain positive effort, d is a positive parameter lower than one to ensure the effort function to be well-behaved (meaning increasing and concave with respect to the real industrial wage), and γ represents the elasticity of industrial real wage to agricultural one. Our specification of $E_{(w_i,w_a)}$ generalizes the one proposed by Akerlof (1982), by opening the additional possibility of having a less that proportional relationship between the wage received by industrial workers, and the wage they would receive if employed in agriculture[22].

Under the above assumptions, and as long as $Wi > W_a$, the FOC for their profit maximization problem imply the Solow condition of unitary wage elasticity of effort (ensuring cost minimization)

$$W_i\ =\ \left(\frac{\omega}{1-d}\right)^{\frac{1}{d}}\ W_a^{\gamma}\ (\underline{P}^*)^{1-\gamma}\,; \tag{7}$$

plus the usual labor demand function

$$L_i\ =\ (1-\beta)^{\frac{1}{\beta}}\ A_i^{\frac{1}{\beta}}\ (E^*)^{\frac{1-\beta}{\beta}}\ K^{\frac{\mu+\beta}{\beta}}\ (W_i)^{-\frac{1}{\beta}}\,; \tag{8}$$

where $E^* \equiv d\omega/(1-d)$ is the effort level corresponding to W_i. Given that the second order conditions are met for the assumed well-behaving production and effort functions, the FOC define the optimal solution as long as the constraint is satisfied.

[21] Several authors have emphasized that efficiency wage mechanisms do not seem appropriate for the rural sector in LDCs, dominated by casual labor and informal relations, while they are much more credible for the formal labor markets of the urban industrial sector. See Mazumdar (1959), Rosenzweig (1988) and Basu (1997), among others.

[22] Note that Akerlof's formalization can be obtained by simply assuming $\gamma = 1$, entailing the perfect proportionality of industrial wages and agricultural ones. Apart from this aspect, the rationality for choosing the above specification is the usual one: the threshold ω is included to avoid the trivial solution of an optimal zero wage (see Akerlof (1982) for more details), and the restrictions on d are needed to ensure the existence of a unique internal maximum.

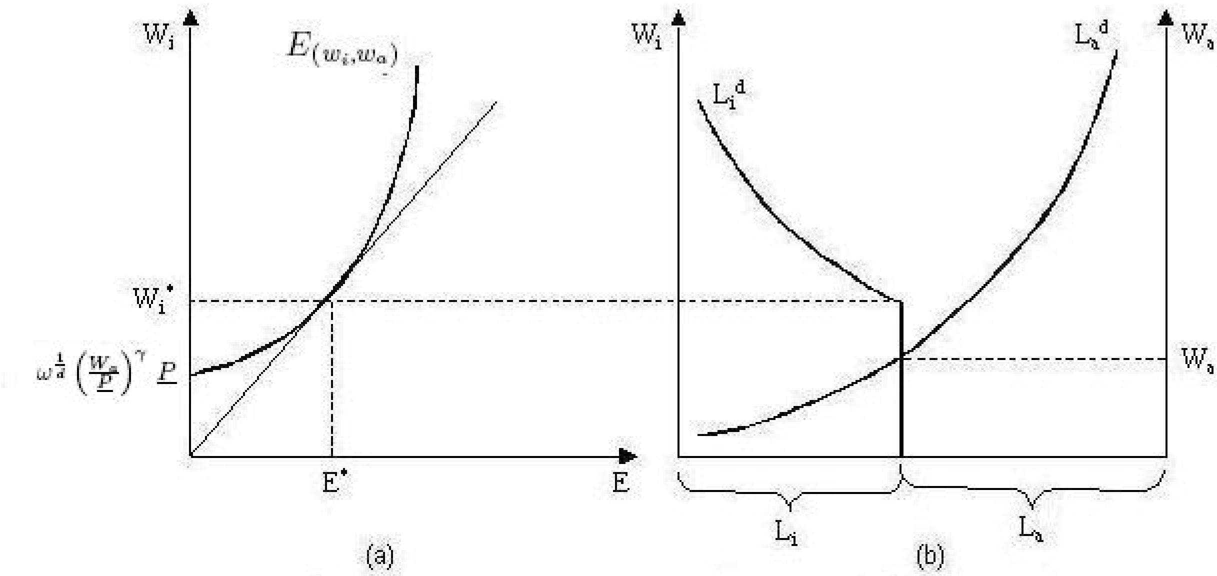

Figure 1: The efficiency wage mechanism

Figure 1(a) represents the diagram corresponding to our specification of effort function on the $W_i - E$ space. The payroll cost per efficiency unit of labor corresponding to each point of the effort function is given by the slope of the ray from the origin to the same point. The optimal wage (indicated in the graph as W_i^*) corresponds to the point of tangency between the ray and the effort function, since the said slope is at its minimum attainable level[23]. Figure 1(b) instead represents the corresponding industrial labor demand on the $W_i - L_i$ space: at W_i^* the labor demand schedule has a kink, because entrepreneurs will resist any wage undercutting and keep the wage at its cost-minimizing level per efficiency unit of labor.

Unless the constraint forces them to act differently, capitalists set the wage at W_i^*, while due to the downward rigidity of the industrial wage, high-earning jobs will be rationed and only L_i^* workers will be hired. The remaining workers will be instead employed in the rural sector at the market clearing wage, in accordance to equation 4 (which determines the L_a^d curve in figure 1b), and consequently a wage gap will arise endogenously across sectors. Clearly, the position of the L_i^d curve depends, among other factors, on the existing stock of capital, with a higher K causing *ceteris paribus* an outwards shift of the curve and hence an increase in L_i.

The adjustment process described so far, follows Kaldor's insights according to which employment creation in the manufacturing sector of typical developing countries is constrained by industrial labor demand and not by supply factors[24]. For this reason, the phase in which $W_a < W_i$ will be called hereafter *Kaldorian underemployment*[25].

[23]It should be noted, however, that the effort function depends on the real agricultural wage ($W_a/\underline{P}$) and on the price index $\underline{P}$, so that the optimal industrial wage itself is increasing in W_a and $\underline{P}$.

[24]Quoting Kaldor's own words (page 386): "... the supply of labour in the high-productivity, high-earning sector is continually in excess of demand, so that the rate of labour-transference from the low to the high-productivity sectors is governed only by the rate of growth of demand for labor in the latter."(1968) See also Kaldor (1967).

[25]Kaldor actually calls this situation "labor surplus", but following Ros (2000) we preferred a different definition, in order to avoid confusion between the notion applied here, and the traditional Lewisian concept of surplus labor. Clearly, the notion of Kaldorian underemployment is logically tied to that of disguised unemployment, but here the mismatch between the shadow wage (that is the opportunity cost of labor outside the modern sector) and the market wage in the industrial sector occurs without any breach of the

The complete analytical description of the inputs' market during the Kaldorian underemployment phase requires to derive also total profits and the labor market clearing, which are respectively given by

$$\Pi = \frac{\beta}{1-\beta} W_i L_i; \tag{9}$$

and

$$L_i + L_a = 1. \tag{10}$$

Note that in the last equation we have normalized the labor force to 1, so that L_a and L_i respectively represent the employment share of the traditional and of the modern sector; this simplifying normalization, however, comes at the cost of eliminating the effect of demographic variables on our economy.

It should be clear at this stage, that Kaldorian underemployment persists only as long as the solution implied by the FOC is admissible, that is as long as $W_a < W_i$. Given the hypothesis of diminishing returns to labor in agriculture, however, the withdrawal of labor from the rural sector is bound to increase W_a; moreover, since the elasticity of industrial wages to rural ones is lower than one, eventually the latter will reach W_i and the constraint will become binding. With reference to figure 1b, the expansion of the industrial sector (a shift of the L_i^d curve toward north-east) tends to close the wage gap, until eventually one uniform wage prevails. Capitalists are then compelled to pay the same wage offered to agricultural workers, and the Kaldorian underemployment phase gives way to the *economic maturity*: "...a state of affairs where real income per head had reached broadly the same level in the different sectors of the economy."[26]. Note also that during the maturity phase employees will be indifferent between working in industry or in agriculture, and thus lack any incentive to increase their effort beyond E^*, despite any possible increase in the uniform real wage rate.

In light of this reasoning, wages in the mature economy will be set at

$$W_i = W_a; \tag{11}$$

while industrial labor demand and total profit will continue being determined by equations 8 and 9, with the only caveat that now the uniform wage rate replaces the value of W_i determined according to efficiency considerations. Obviously, the rural wage and rents determination, and the labor market clearing (equation 4, 5 and 10 respectively) will hold also during maturity.

MARKET CLEARING

The complete characterization of the economy involves one more equation related to the balance of payment, and prescribing the equilibrium of the trade balance[27]. Assuming that a constant proportion s of total profits is reinvested, while wage income as well as rents are entirely consumed, the balance of payment equilibrium can be expressed by

$$P_a^* \left(X_a^c - X_a^s\right) + \left(X_i^c + s\,\Pi - X_i^s\right) = 0; \tag{12}$$

DYNAMIC OF CAPITAL STOCK

marginal theory of distribution.

[26]The quotation is Kaldor's own definition of economic maturity, which he also defined as "the end of the dual economy" (1968).

[27]More precisely, the equilibrium of the balance of payment would require the trade balance to match the exogenously determined net inflow of resources recorded in the capital and financial accounts. In the long run, however, there is no reason to assume a systematically positive or negative inflow of capital, and accordingly, we set such value at zero.

As concerns the dynamic of the state variable K (hence the long run characterization of the economy), let us suppose that the rate of accumulation depends positively - through a generic monotonic function $g(.)$ - on the domestic profit rate Π/K relative to the exogenously determined international risk-adjusted profit rate r^*. This idea is captured algebraically by

$$\hat{K} = g\left(\frac{\Pi}{K} - r^*\right); \qquad g'(.) > 0; \qquad g(0) = 0. \tag{13}$$

and allows to take into account both adjustment costs, and the presence of international capital mobility.

Stated as it is, ours is a "supply-limited model of industrial growth" - using Taylor's 1988 jargon - with market-clearing prices and flexible capital labor ratio. It is important to emphasize that the choice of a supply-limited model in this context is not meant to undervalue the importance of keynesian arguments concerning the level of effective demand, but only to focus our attention on the *potential* growth path of an economy. Apart from the presence of increasing returns in industry, the distinctive feature of this model is the dualistic characterization of the labor market, leading to a Keynesian-like adjustment of the labor market, in which demand is the driving force of sectoral labor allocation.

THE EQUILIBRIUM CONFIGURATION

Rather than directly solving the whole system of equations and determine the steady states, we prefer to proceed in three stages to highlight the various economic mechanisms at work in the development process. Holding the capital stock as a pre-determined variable - hence in the short run - the economy is analytically described by a system of eleven independent equations with eleven endogenous variables (X_a^c, X_i^c, X_a^s, X_i^s, L_a, L_i, W_a, R, W_i, E, Π). It is thus possible to determine the nominal industrial wage consistent with the clearing of the goods' market for each given level of capital stock; hereafter the corresponding locus of short-run equilibria in the $\log W_i - \log K$ space is called **product wage schedule** (indicated as PW)[28]. At a second stage, the **locus of stationary capital stock** can be obtained from the dynamic equation 13, to express the value of the nominal industrial wage corresponding to a null net investment. Finally, from the dynamic equation one can determine the necessary and sufficient conditions for the existence of steady state equilibria and for their stability properties. Clearly, because of the dichotomic working of the labor market before and after the maturity threshold $W_a = W_i$, the two *loci* shall be derived separately for the two phases.

As emphasized by the "high development theory", the elasticity of industrial labor supply is the pivotal magnitude summarizing the economic mechanisms at work. Its crucial role stems from the fact that in two-sectors macro models - unlike in aggregate models - this elasticity depends on the interaction between technological conditions (namely the evolution of labor productivity across sectors), demographic variables[29], and movements in relative prices, while it concurs to determine the speed of labor reallocation across sector, and the effect of such reallocation in terms of profitability.

During Kaldorian underemployment, the eleven equations composing the system are: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12. Interestingly, the structure of causation in the model is such that the equations related to the labor market and distributional issues (equation number 4, 5, 6, 7, 8, 9 and 10) actually

[28]Recall that through an appropriate choice of unit we have normalized P_i^* to one, so that in industry the nominal wage equals the product wage.

[29]The influence of demographic variables and more generally of the demographic transition certainly plays an important role in the development trajectory of poor countries, as is evident from the attention paid to birth control in countries like China or India. Without disregarding the importance of demographic factors, we decided however to overlook this aspect (see the normalization adopted in equation 10) and concentrate on different mechanisms more directly related to the structural modification of the economy. Given the dualistic working of the labor market, one can argue that in any case a positive growth of the labor force would imply a longer phase of Kaldorian underemployment, since new-born workers would be employed in the primary sector thereby postponing the advent of maturity and the prevalence of a uniform wage.

constitute a sub-system that jointly determines L_a, L_i, W_a, R, W_i, E and Π. Only at a second stage, by means of the sectoral production functions (equation 2 and 3), supplied quantities are also derived; and finally the remaining equations (number 1 and 12) determine the consumption levels of each good[30].

As shown in Mathematical Appendix I, after some algebraic manipulations of the subsystem mentioned above one obtains the elasticity of labor supply (ϵ_{ku}^{LS}) faced by industrial entrepreneurs in the Kaldorian underemployment phase

$$\epsilon_{ku}^{LS} \equiv \frac{\partial \log L_i}{\partial \log W_i} = \frac{1 - L_i}{\gamma\, b L_i}; \tag{14}$$

which is valued in the interval $[0, +\infty)$ [31]. Observe further that ϵ_{ku}^{LS} is a decreasing function of L_i, given that a higher industrial labor share leads, *ceteris paribus*, to higher agricultural wages, and hence W_i will have to grow proportionally more to attract additional workers to industry.

Continuing with a bit of algebra (see the Mathematical Appendix I), it can be demonstrated that during Kaldorian Underemployment the equation of the product wage schedule in log terms is given by

$$\log \Theta - \log A_a + \frac{1}{\gamma} \log W_i + b \log \left[1 - A_i^{\frac{1}{\beta}} \Phi \exp\left(\frac{\mu + \beta}{\beta} \log K - \frac{1}{\beta} \log W_i \right) \right] = 0; \tag{15}$$

where Θ and Φ are constants defined respectively as

$$\Phi \equiv (1 - \beta)^{\frac{1}{\beta}} (E^*)^{\frac{1-\beta}{\beta}}; \qquad \Theta \equiv \frac{1}{1 - b} \left(\frac{1 - d}{\omega} \right)^{\frac{1}{d\gamma}} (\underline{P}^*)^{-\frac{1-\gamma}{\gamma}} (P_a^*)^{-1}.$$

As attains the long-run equilibrium of the system, instead, the hypotheses made on function $g(.)$ in equation 13 are such that net investment is zero when the domestic profit rate equals the international risk-adjusted profit rate, that is for

$$\frac{\Pi}{K} - r^* = 0.$$

To derive the expression of the stationary capital locus on the $\log W_i - \log K$ plane, simply replace Π in the above condition with its short-run equilibrium value, given during Kaldorian underemployment by equations 8 and 9. Expressing everything in logarithmic terms, this operation yields after some manipulations

$$\log \frac{\beta\, \Phi}{1 - \beta} + \frac{1}{\beta} \log A_i - \frac{1 - \beta}{\beta} \log W_i^{**} + \frac{\mu}{\beta} \log K = 0; \tag{16}$$

where we used the notation W_i^{**} in order to distinguish the wage compatible with break-even investment from the short-run equilibrium wage.

Total differentiation of equation 15 yields the coefficient of the product wage schedule for the Kaldorian underemployment interval:

$$\frac{\partial \log W_i}{\partial \log K} = \frac{\mu + \beta}{1 + \beta \epsilon_{ku}^{LS}}. \tag{17}$$

This coefficient is surely positive, given that the labor supply elasticity is non-negative, and it is decreasing in ϵ_{ku}^{LS}[32]. Indeed, a given increase in the capital stock will trigger an outflow of labor from

[30] Note that, because of the structure of causality, the sub-system at issue is actually sufficient to determine the steady state equilibria, while the last two steps are only relevant for a full characterization of the equilibria previously defined.

[31] As expected given the structure of causality of the model, the industrial labor supply elasticity does not depend on Z, nor on the non-homoteticity of the preferences' structure, but only on variables underlying the labor market.

[32] Note that the PW schedule is hence convex in the $\log W_i - \log K$ plane, since the labor supply elasticity is decreasing in $\log K$.

agriculture[33], and the higher the elasticity of industrial labor supply the smaller - *ceteris paribus* - the adjustment in nominal industrial wages required by the expansion L_i. Besides, since a raise in industrial employment share reduces ϵ_{ku}^{LS}, the product wage schedule will be flatter for low levels of L_i, and get gradually steeper as industry expands its employment basin. On the other hand, the higher the output elasticity to capital $(\mu + \beta)$, the higher the industrial wage in equilibrium, hence the greater the coefficient of the product wage schedule.

As for the stationary capital locus, a close inspection of equation 16 shows that in the $\log W_i - \log K$ space it represents a straight line sloped

$$\frac{\partial \log W_i^{**}}{\partial \log K} = \frac{\mu}{1-\beta}. \tag{18}$$

Given the parametrization, the coefficient is positive and increasing in μ: the higher the external capital effect, the stronger the positive impact of capital accumulation on the industrial TFP, the higher total profits and the higher the nominal wage compatible with the break-even level of investment. On the other hand, the stationary capital locus is also steeper the greater the capital share, because a higher β means, *ceteris paribus*, a higher level of total profits for the same increase in capital stock, so a higher level of reinvestment.

In plain words, during Kaldorian underemployment higher values of the capital stock trigger the expansion of industries (in terms of both labor share and output), leading to a raise of the rural wage, which in turn drives the upwards adjustment of industrial wages to satisfy the Solow condition. As shown in Mathematical Appendix I, the adjustment process required to get the equilibrium in the goods' market is such that higher levels of K entail a reduction in the wage (and productivity) gap between manufacturing and agricultural activities, to the extent that for sufficiently high capital stock a unique uniform wage (and labor productivity) will prevail in the economy. It is worth noting the process of labor reallocation just described is Pareto improving, and entails a double gain in terms of growth favored also by the unfold of increasing returns in manufacturing[34]. Additionally, such process rationalizes several stylized facts often cited in the literature concerning LDCs[35]: (i) the "agriculture-industry shift"; (ii) the existence of wide productivity gaps across economic sectors, with agriculture featuring a much higher employment share than its correspondent GDP share, and hence having a lower average labor productivity than the rest of the economy[36]; (iii) the progressive reduction of intersectoral productivity (and wage) gaps, as labor reallocation raises agricultural labor productivity relative to the rest of the economy.

Once the rural wage reach their counterpart in manufacturing, the constraint to the entrepreneurs' profit maximization $(W_a = W_i)$ becomes binding, the system enters the maturity phase and the above equilibrium configuration ceases to hold. Indeed, the short-run characterization of the mature economy is still described by equations 1, 2, 3, 4, 5, 6, 8, 9, 10 and 12, but unlike in the Kaldorian underemployment phase equation 11 now replaces equation 7. As shown formally in Mathematical Appendix II, the prevalence of one uniform wage alters the industrial labor supply elasticity, which is then equal to

$$\epsilon_{ma}^{LS} = \frac{1-L_i}{bL_i}. \tag{19}$$

[33] Industrial labor demand depends positively on the capital stock K (see equation 8).

[34] Again Kaldor (1968) expresses this idea very clearly: "... the growth of productivity is accelerated as a result of the transfer at both hands - both at the gaining end and at the losing end; in the first, because, as a result of increasing returns productivity in industry will increase faster, the faster output expands; in the second because when the surplus-sectors lose labour, the productivity of the remainder of the working population is bound to rise."

[35] For a more detailed exposition of these stylized facts see among others Kuznets (1966), Chenery and Syrquin (1975), Syrquin (1989), Taylor (1989) and Bhaduri (1993 and 2003).

[36] As pointed out by the Todarian theory of migration, such productivity gaps are mirrored by urban-rural wage gaps, which act as a stimulus to labor reallocation toward city-based industrial employment; see Todaro (1969) and Harris and Todaro (1970).

Similarly to the previous phase, ϵ_{ma}^{LS} is non-negative and decreasing in the industrial labor share.

Correspondingly, the uniformity of the wage across sectors modifies the PW schedule into

$$-\log(1-b)\,P_a^* \; - \log A_a \; + \; \log W_i \; + \; b \log\left[1 \; - \; A_i^{\frac{1}{\beta}}\,\Phi\,\exp\left(\frac{\mu+\beta}{\beta}\log K - \frac{1}{\beta}\log W_i\right)\right] \; = \; 0. \quad (20)$$

Total differentiation shows that the curve on the usual $\log W_i - \log K$ plane is sloped

$$\frac{\partial \log W_i}{\partial \log K} = \frac{\mu+\beta}{1+\beta\epsilon_{ma}^{LS}}; \quad (21)$$

and that it continues to be convex even during maturity (for a formal proof see the Mathematical Appendix II)[37]. The prevalence of a uniform wage steepens the short-run equilibrium locus compared to the Kaldorian underemployment phase, however the industrial labor share can gradually increase with capital accumulation even in the mature phase, due to the possibility opened by international trade[38].

As for the long-run equilibrium of the system, even in maturity the stationary capital locus will continue to be expressed by equation 16, given that equation 8 continues to hold and that nothing alters the differential equation of capital accumulation (13).

Diagrammatically, we can determine the equilibria and their stability properties, by superimposing the short- and long-run equilibrium loci, and analysing respectively the interception points, and the relative position of the two curves. Ideally, the economy moves along the product wage diagram, with the capital stock growing as long as the short-run equilibrium wage lies below the $\hat{K} = 0$ locus, and shrinking if the opposite happens. The reason for this is the behavior of total profits, and hence of investment: when the short-run equilibrium wage lies below that compatible with null net investment, reinvested profits will exceed depreciation costs and fuel capital accumulation, while in the opposite situation net investment will be negative and capital stock will fall.

From an analytical point of view, the differential equation 13 can be rewritten replacing Π with its short-run value from equation 8 and 9, to obtain

$$\hat{K} \; = \; g\left[\frac{\beta}{1-\beta}\,A_i^{\frac{1}{\beta}}\,\Phi\,\exp\left(\frac{\mu}{\beta}\log K - \frac{1-\beta}{\beta}\log W_i\right) \; - \; r^*\right]. \quad (22)$$

Unless specifying the function $g(.)$ and obtaining an explicit solution for $\log W_i$ in terms of $\log K$, it is clearly impossible to fully characterize the dynamic behavior of the model. Nevertheless, it is possible, given the monotonicity of $g(.)$, to infer the trend of $\hat{K}$ by studying the sign of its first and second derivatives with respect to $\log K$. One should keep in mind, however, that this procedure is far from obtaining a complete mapping of the function.

As for the first derivatives under question, it is given by

$$\frac{\partial \hat{K}}{\partial \log K} = g'(.)\,\frac{1}{1-\beta}\,\frac{W_i\,L_i}{K}\,\left[\mu - (1-\beta)\,\frac{\mu+\beta}{1+\beta\,\epsilon^{LS}}\right];$$

[37]Thus, regardless of the phase of the economy, one can always express the slope of the product wage schedule as

$$\frac{\partial \log W_i}{\partial \log K} \; = \; \frac{\mu+\beta}{1+\beta\epsilon^{LS}}$$

with the only caveat that ϵ^{LS} is not everywhere continuous, but falls abruptly once the system enters into maturity. While being a piece-wise function, the PW schedule is continuous over its whole domain, and continuously differentiable but with the exception of the corner point.

[38]In the closed economy, instead, the industrial labor share stabilizes once the system enters in the maturity phase, and successive rises in the capital stock do not trigger any further reallocation of workers across sectors. See Valensisi (2008).

[39]which is positive for

$$\epsilon^{LS} > \frac{1-\beta-\mu}{\mu}; \tag{23}$$

and negative otherwise, due to the sign restriction on $g'(.)$. Given the range of values possibly taken by the labor supply elasticity, the above condition is surely satisfied for a non-empty interval corresponding to the highest values of ϵ^{LS} (the lowest value of capital stock). In light of this, it is possible to apply De L'Hospital's rule to equation 22 and show that $lim_{K\to 0}\,\hat{K} = g(-r^*) < 0$. Additionally, the same theorem can be used to prove that for K tending to infinity, $\hat{K}$ tends to $g(-r^*) < 0$ whenever $\mu < 1+\beta$, it converges to $g(A_i^{\frac{1}{\beta}}\,\Phi - r^*)$ when industry uses a AK technology ($\mu = 1+\beta$), and diverges to infinite for $\mu > 1+\beta$ [40].

Concerning the second derivative of $\hat{K}$ with respect to $\log K$, its sign cannot be determined *a priori*, unless specifying the function $g(.)$[41]. Even without specifying the said function, it can nevertheless be shown that for $\epsilon^{LS} = (1-\beta-\mu)/\mu$,

$$\frac{\partial^2 \hat{K}}{\partial \log K^2} = \frac{W_i L_i}{K}\, g'(.)\, \frac{\beta\,(\mu+\beta)}{(1+\beta\,\epsilon^{LS})^2}\,\frac{\partial \epsilon^{LS}}{\partial \log K} < 0;$$

proving that $\hat{K}$ has a local maximum in the corresponding point[42]. Note however that lacking an explicit formalization of g(.) one cannot directly solve for the maximum of $\hat{K}$, and hence it is not possible to dismiss on theoretical grounds the possibility of $\hat{K}$ being negative even at its maximum value, in which situation no matter how big the capital stock, industrialization would never be self-financing[43].

Apart from the extreme outcome of an economy being always stuck at purely agrarian state, in light of the above we can summarize our results into proposition I.

Proposition I *The necessary and sufficient condition for the existence of an unstable low development equilibrium with positive K (inequality 23) are surely met for a sufficiently low capital stock, meaning that the equilibrium with a purely agrarian economy is always locally stable. On the other hand, the necessary and sufficient condition for the existence of a stable equilibrium of full industrialization require decreasing returns to accumulable factors, and hence*

$$\mu < 1-\beta. \tag{24}$$

Proposition I derives from the fact that condition 23 ensures the existence of an interval in which $\hat{K}$ is increasing with respect to $\log K$, while condition 24 guarantees the existence of an interval in which

[39]Note that in computing the derivative of $\hat{K}$ with respect to $\log K$ we made use of the result mentioned in footnote 37.

[40]Note that in order to solve the indeterminacy of the limit of $\hat{K}$ for K and W_i tending to zero (or to infinity) one needs to rewrite equation 22 as

$$\hat{K} = g\left[\frac{\beta}{1-\beta}\,A_i^{\frac{1}{\beta}}\,\Phi\,K^{\left(\frac{\mu}{\beta}-\frac{1-\beta}{\beta}\frac{\log W_i}{\log K}\right)} - r^*\right];$$

(transformation that is legitimate but for $K = 1$), and then, since $\hat{K}$ is continuous but for $K = 1$, De L'Hospital's theorem can be applied to solve the indeterminate form $\log W_i/\log K$.

[41]algebraically, the derivative under question is given by

$$\frac{\partial^2 \hat{K}}{\partial \log K^2} = \frac{1}{1-\beta}\frac{W_i L_i}{K}\left[\mu-(1-\beta)\frac{\mu+\beta}{1+\beta\epsilon^{LS}}\right]^2\left(g''(.)\frac{1}{1-\beta}\frac{W_i L_i}{K}+g'(.)\frac{1}{\beta}\right)+\frac{W_i L_i}{K}g'(.)\frac{\beta\,(\mu+\beta)}{(1+\beta\epsilon^{LS})^2}\frac{\partial\epsilon^{LS}}{\partial \log K}.$$

[42]To demonstrate the above result, recall that ϵ^{LS} is decreasing in L_i, and hence - being L_i a monotonic increasing function of $\log K$ - the derivative $\partial\epsilon^{LS}/\partial \log K$ is surely negative.

[43]A closer analysis reveals that such hardly plausible outcome results from extremely low values of the industrial TFP relative to ω (the threshold level of real wage necessary to obtain positive effort from workers) and to the international risk-adjusted profit rate r^*.

$\hat{K}$ is decreasing with respect to $\log K$. Hence under the plausible assumption that there exists of an interval in which $\hat{K}$ is positive, De L'Hospital's theorem yields sufficient conditions for our results. Proposition I can easily be interpreted in terms of slopes of PW and $\hat{K} = 0$ locus: it simply states that for a stable (unstable) equilibrium to occur the product wage schedule should be steeper (flatter) than the stationary capital arm, and cut is from below (above).

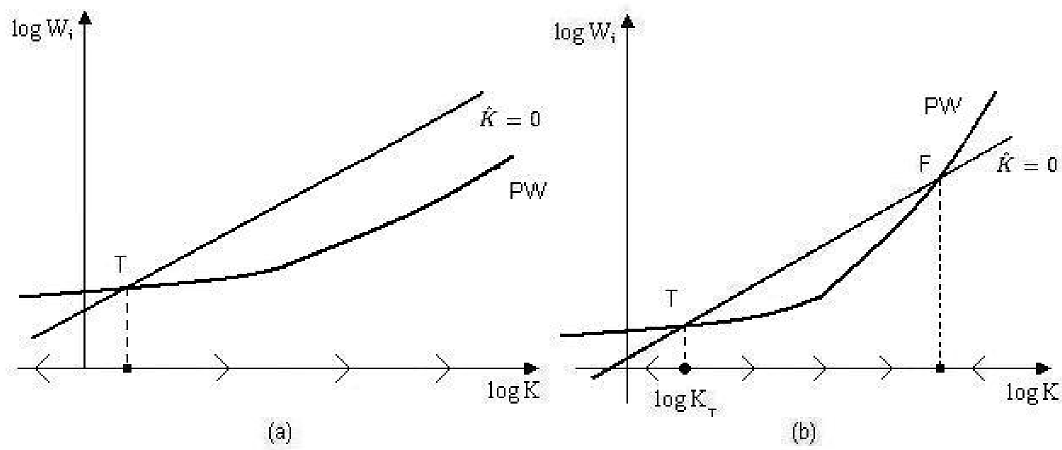

Figure 2: The model

Figure 2 presents the two possible configurations of the system, which are characterized by different parametrizations. The third alternative, which even if trivial cannot be dismissed on theoretical grounds, is the one in which no matter how big the initial capital stock, industrialization would never be self-financing, and the economy will always need to finance its imports of manufactures by exporting primary agricultural commodities.

Consider first the case of figure 2a, in which $\mu \geq 1-\beta$ (with equality yielding an AK technology) and $\hat{K}$ increases with respect to $\log K$ over the whole domain. In such a case two possible equilibria arise: a locally stable equilibrium with zero capital and a fully agrarian economy, and an unstable equilibrium T, where both sectors coexist. Clearly, for $K < K_T$ the system is in the basin of attraction of the low development equilibrium: industrialization is not viable and the economy specializes in food production while trading some primary commodities in exchange for foreign manufactures. On the other hand, for capital stock greater than K_T, the specialization in manufactures takes place profitably, and can proceed indefinitely (in the case of AK technology) or even at a growing speed (for μ strictly greater than $,1-\beta$).

Alternatively, consider the case illustrated in figure 2b, where $\hat{K}$ is first increasing and then decreasing in $\log K$. Three equilibria are then possible: (i) a locally stable equilibrium of pure subsistence with zero capital stock, (ii) an unstable low development equilibrium at point T, and (iii) a stable equilibrium of full industrialization at F. For capital stocks lower than K_T there is an unstable poverty trap causing capital stock to shrink over time while the economy falls back to the state of pure agricultural producer. On the other hand, when $K > K_T$ the effect of increasing returns raises profitability enough to trigger an accelerated growth and a self-fulfilling process of capital accumulation, driving the system to the equilibrium of full industrialization F in which the primary and the manufacturing sector coexist.

Figure 2 shows that even if the economy is open to international trade, its capital stock may be too low to sustain the expansion of the industrial base and this fact locks in the specialization in the production of primary commodities. This situation may call for a big push à la Rosenstein Rodan, that is a concerted investment capable of bringing the capital stock beyond K_T, breaking the poverty trap and making the industrialization process feasible[44]. We mention in passing that the relevance of coordination failures and big push policies, even when the economy is open to international trade, is consistent with models in which basic intermediate goods (such as infrastructures, and other "social overhead capital") are produced under increasing returns[45].

In light of the recent wave of criticism against the idea of poverty traps[46], few words should be spent commenting the situations described in Figure 2. First of all, it should be pointed out that the poverty trap discussed here is not driven by lack of savings, but by insufficient profitability. Increases in the saving propensity do not matter for the existence of the poverty trap, but only reduce the basins of attraction of the zero capital equilibrium.

Secondly, the unstable equilibrium of pure agrarian economy does not necessarily entail a zero growth: the analysis so far has taken sectoral TFP as parameters, however exogenous technical progress acts also in the agricultural sector, and may spur the growth performances even of a completely agricultural economy (in addition to modifying the whole equilibrium configuration, as will be shown later). Thirdly, it is worth noting that the degree of increasing returns required to make the poverty trap a relevant case in our set-up is far lower than in other aggregate models; even a value of μ around 0.1 (hence within the estimates cited by Kraay and Raddatz) are sufficient to generate the low equilibrium trap. The reason is that the effect of increasing returns is amplified here by the elasticity of industrial labor supply, a factor rather disregarded in aggregate models of growth, though crucial here.

Finally we note in passing that the above model suggests a theoretical mechanism that traces the multiplicity of equilibria to the structural characteristics of the economy, namely the extent of "agriculture-industry shift". Simulated work based on analogous premises (the "variable returns to scale model") has recently confirmed that this line of reasoning may be empirically fruitful in explaining the poor economic performance of LDCs vis à vis rich nations[47]. Moreover, such link is consistent with the empirical findings on cross-country growth regressions with parameters' heterogeneity, which point out how poorest countries typically seem to follow a structurally different growth regime than middle- and high-income nations [48].

III. *Comparative statics and the Dutch disease*

PARAMETRIC INCREASE IN AGRICULTURAL TFP

So far, the analysis of the two-sectors economy abstracted from technical progress, and treated the sector-specific TFPs as exogenous parameters. This approach may be convenient from an analytical point of view, but overlooks one of the main forces - if not the main force - behind the long-term increases in income: technical change.

Needless to say, increases in TFP, be it agricultural or industrial, have an unambiguous positive welfare effect, for they allow a greater supply of goods by using more efficiently the given amount of

[44] We remind here that in our set-up K includes by definition also the effect of embodied knowledge; as a consequence, the interpretation of big push policies should be very careful, since only in so far as the investment process also raises the knowledge base do our conclusions hold.

[45] This element is emphasized for instance by Ros an Skott (1997), Hoff (2000) and Sachs (2005). Despite its relevance, the big push argument should be considered with caution, and not uncritically equated to the so-called "classical aid narrative" (see Easterly 2006), which claims that a sufficient amount of aid would automatically lift countries out of the poverty trap to the take off.

[46] See Kraay and Raddatz 2007 and Easterly 2006.

[47] See Graham and Temple (2005).

[48] See Durlauf and Johnson (1995); Durlauf, Kourtellos and Minkin (2001) and Durlauf, Johnson and Temple (2005).

resources. More complex, however, are the effects of technical progress on the equilibrium configuration for the whole dynamic system. Precisely to grasp these effects, we now carry out some comparative statics exercises with regard to sectoral TFPs. It is worth repeating here that in the case of the agricultural sector A_a summarizes the effect not just of technical progress but also of geographical and climatic conditions, so that an increase in A_a may also result from temporary favorable conditions.

As seen before, any long-run equilibrium, whether stable or unstable, is basically defined by the system between the relevant expression for the product wage schedule (equation 15 for Kaldorian Underemployment and equation 20 for the maturity phase) and the stationary capital locus (equation 16 in the whole domain). To lean down the notation let us rewrite the system as

$$\begin{cases} PW(\log W_i, \log K, A_a, A_i) = 0; \\ G(\log W_i, \log K, A_a, A_i) = 0; \end{cases} \tag{25}$$

where the implicit function PW (.) is the short-run equilibrium schedule and G(.) indicates the stationary capital locus.

Besides, recall that the product wage schedule is continuously differentiable with respect to its four arguments (but with the exception of the corner point corresponding to the threshold between Kaldorian Underemployment and maturity), while the $\hat{K} = 0$ locus is continuously differentiable with respect to the four arguments on its whole domain. In light of this, and provided that the Jacobian of system 25 is non singular, the hypotheses underlying the implicit function theorem are satisfied over the whole domain, excluding the neighborhood of the corner point. With such exception, the theorem can therefore be applied in the neighborhood of a generic equilibrium (call it point Q) to rewrite system 25 as

$$\begin{cases} PW(\log W_i^Q(A_a, A_i), \log K^Q(A_a, A_i), A_a, A_i) = 0; \\ G(\log W_i^Q(A_a, A_i), \log K^Q(A_a, A_i), A_a, A_i) = 0; \end{cases} \tag{26}$$

in which ($\log W_i^Q$,$\log K^Q$) are the coordinates of the equilibrium point.

As concerns changes in the agricultural total factor productivity, the chain rule theorem can be used to compute the total derivative of each function in system 26 with respect to A_a, obtaining:

$$\begin{cases} \left.\frac{\partial PW}{\partial \log W_i}\right|_Q \frac{\partial \log W_i^Q}{\partial A_a} + \left.\frac{\partial PW}{\partial \log K}\right|_Q \frac{\partial \log K^Q}{\partial A_a} = -\frac{\partial PW}{\partial A_a}; \\ \left.\frac{\partial G}{\partial \log W_i}\right|_Q \frac{\partial \log W_i^Q}{\partial A_a} + \left.\frac{\partial G}{\partial \log K}\right|_Q \frac{\partial \log K^Q}{\partial A_a} = -\frac{\partial G}{\partial A_a}. \end{cases} \tag{27}$$

Solving this last system for $\partial \log W_i^Q / \partial A_a$ and $\partial \log K^Q / \partial A_a$ permits to obtain, from the sign of these derivatives, the direction in which the new equilibrium value (call it Q') resulting from the change in A_a will lie. Analytically, it can be shown that such solutions are[49]

$$\frac{\partial \log W_i^Q}{\partial A_a} = \frac{\begin{vmatrix} -\frac{\partial PW}{\partial A_a} & \frac{\partial PW}{\partial \log K} \\ -\frac{\partial G}{\partial A_a} & \frac{\partial G}{\partial \log K} \end{vmatrix}}{|J|}; \qquad \frac{\partial \log K^Q}{\partial A_a} = \frac{\begin{vmatrix} \frac{\partial PW}{\partial \log W_i} & -\frac{\partial PW}{\partial A_a} \\ \frac{\partial G}{\partial \log W_i} & -\frac{\partial G}{\partial A_a} \end{vmatrix}}{|J|}; \tag{28}$$

While these two expressions hold in general over the whole domain (except in the neighborhood of the corner point), the piecewise nature of the product wage schedule implies that comparative statics should be carried out separately for each phase: Kaldorian underemployment and maturity.

Proceeding with a taxonomic logic, suppose first that equilibrium Q occurs during the Kaldorian underemployment phase. In such a case, the partial derivatives in 28 should be replaced with their

[49] Of course in the following section all partial derivatives should be valued at Q, that is at the value corresponding to the equilibrium; for simplicity we omit this detail from the notation of the text.

actual values computed from equations 15 and 16. Indicating with J^{KU} the Jacobian corresponding to the Kaldorian underemployment phase, this operation yields

$$\frac{\partial \log W_i^Q}{\partial A_a} = \frac{\mu}{\beta A_a} \frac{1}{|J^{KU}|}; \qquad \frac{\partial \log K^Q}{\partial A_a} = \frac{1-\beta}{\beta A_a} \frac{1}{|J^{KU}|}; \tag{29}$$

implying under the assumed parametrization that the two derivatives under consideration assume the same sign of $\left|J^{KU}\right|$ (see Mathematical Appendix III.A for more details).

Moving to the maturity phase, the same procedure shall be followed to carry out the comparative statics, replacing the partial derivatives of equation 28 with their actual values calculated from equations 20 and 16. As shown in Mathematical Appendix III.A, this procedure yields

$$\frac{\partial \log W_i^Q}{\partial A_a} = \frac{\mu}{\beta A_a} \frac{1}{|J^{MA}|}; \qquad \frac{\partial \log K^Q}{\partial A_a} = \frac{1-\beta}{\beta A_a} \frac{1}{|J^{MA}|}; \tag{30}$$

proving that during maturity the two derivatives considered take the opposite sign of $\left|J^{MA}\right|$.

Furthermore, Samuelson's "correspondence principle between statics and dynamics"[50] can be utilized to prove that

$$\left|J^{KU}\right| > 0 \quad \Longleftrightarrow \quad \mu > \frac{1-\beta}{1+\epsilon_{ku}^{LS}};$$

and

$$\left|J^{MA}\right| > 0 \quad \Longleftrightarrow \quad \mu > \frac{1-\beta}{1+\epsilon_{ma}^{LS}};;$$

meaning that $\left|J^{KU}\right|$ $\left(\left|J^{MA}\right|\right)$ is positive when the corresponding equilibrium point is unstable, and negative in the opposite case[51]. The implications of the correspondence principle for equation 29 and 30 can thus be summarized in proposition II.

Proposition II *Parametric increases in agricultural TFP create a situation of Dutch disease*[52]*: they enlarge the basin of attraction of the locally stable equilibrium of purely agricultural economy, and move the stable equilibrium of full industrialization (if any) towards South-West, lowering the steady state values of* $\log W_i$ *and* $\log K$.

The results of proposition II are shown diagrammatically in figure 3, representing the case in which there is also a stable equilibrium of full industrialization[53] (dashed schedules represent the equilibrium locus before the TFP increase). The modification of the product wage schedule, vis à vis the invariance of the stationary capital locus, widens the basin of attraction of the low-level equilibrium - from $(-\infty, \log K^T)$ to $(-\infty, \log K^{T'})$ - correspondingly rising the minimum critical level of capital beyond which increasing returns make industrialization self-sustaining. Moreover, the rise in agricultural TFP tends to counteract the potential specialization of the economy in manufactures, thereby reducing the steady state wage, capital stock and industrial labor share, whenever the asymptotically stable equilibrium of full industrialization exists.

The Dutch disease emerges here because the increase in A_a favors the specialization of the economy in primary commodities: by increasing the wage to be paid to attract workers to the industrial sector

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[50]The principle was analyzed by Samuelson in 1941 and 1947; for a recent treatment of the principle see Gandolfo (1997).

[51]Recall that for the implicit function theorem to hold, $\left|J^{KU}\right|$ and $\left|J^{MA}\right|$ must be different from zero.

[52]In line with the existing literature, the expression Dutch disease refers here to a situation in which the boom in agricultural productivity (or more generally in natural-resource-intensive sectors) crowds out the manufacturing sector. Clearly such phenomenon can occur through changes in the comparative advantages as in the case Graham's paradox (1923) and Matsuyama (1992) or through appreciations of the real exchange rate as in Krugman (1987) and Ros (2000).

[53]Clearly had the situation been that of figure 2a, the only change would have been on the unstable equilibrium T.

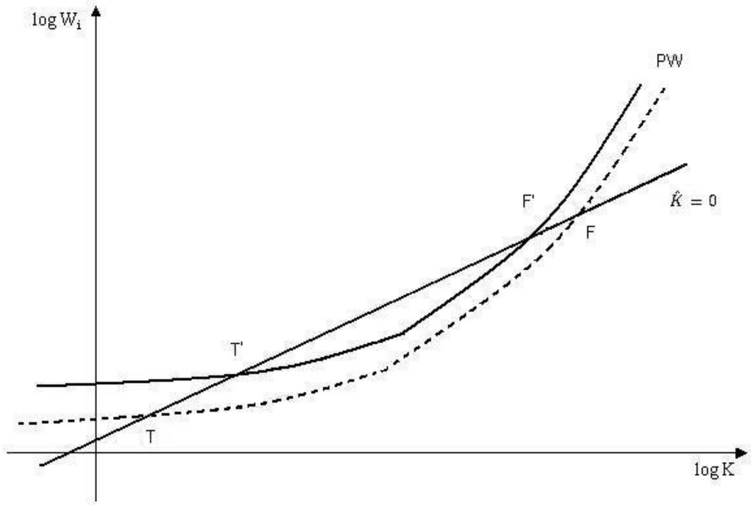

Figure 3: The effect of an increase in agriculture TFP

(be it during Kaldorian underemployment or during maturity), this parametric change represents a burden to the unfold of increasing returns in manufacturing. This outcome reflects the fact that the food and manufacturing sectors compete for the same workers, with the two wages being tied to each other through efficiency wage mechanisms, but - unlike in the close economy set-up - they do not have any complementary role[54]. Indeed, despite the presence of Engel's effect in demand composition, the prevalence of international prices blurs away any positive feedback between increased food availability and a higher income share devoted to manufacturing consumption, feedbacks that would favor domestic industries had the economy been close to international trade. Here, by mean of international trade the increase in agricultural TFP affects only the sectoral labor allocation, and possibly the balance of payment structure leaving unchanged the margin of profitability of the entrepreneurs. The seemingly paradoxical outcome that growth in agricultural TFP has a positive effect on industrialization in a closed economy but a negative effect once the country is open to trade confirms the results of similar models of structural change (see Matsuyama 1992). The peculiarity of the present approach in modeling labor market interactions, however, generalizes the analysis by including the additional case in which multiple equilibria arise and a country may be caught in a poverty trap.

THE CASE OF PARAMETRIC INCREASE IN INDUSTRIAL TFP

Applying the same procedure used for parametric changes in the agricultural TFP, we can shed some light also on the comparative statics regarding increases in A_i [55]. Total derivation of system 26 with

[54]See Valensisi (2008) for the closed-economy version of this model, in which a higher productivity in the food sector actually acts as a stimulus to agriculture, reducing the basin of attraction of the low equilibrium (if any) and increasing the steady state value of K and W_i for the stable equilibrium of full industrialization (if any).

[55]Note that, because of the algebraic properties of Cobb Douglas production functions, all forms of technical change - unbiased, labor augmenting and capital augmenting (also called Hicks neutral, Harrod neutral and Solow neutral) - translate into variations of the parameter A_i, and are thus essentially indistinguishable from one another.

respect to A_i yields

$$\begin{cases} \left.\frac{\partial PW}{\partial \log W_i}\right|_Q \frac{\partial \log W_i^Q}{\partial A_i} + \left.\frac{\partial PW}{\partial \log K}\right|_Q \frac{\partial \log K^Q}{\partial A_i} = -\frac{\partial PW}{\partial A_i}; \\ \left.\frac{\partial G}{\partial \log W_i}\right|_Q \frac{\partial \log W_i^Q}{\partial A_i} + \left.\frac{\partial G}{\partial \log K}\right|_Q \frac{\partial \log K^Q}{\partial A_i} = -\frac{\partial G}{\partial A_i}; \end{cases} \tag{31}$$

which can be solved for $\partial \log W_i^Q / \partial A_i$ and $\partial \log K^Q / \partial A_i$ obtaining

$$\frac{\partial \log W_i^Q}{\partial A_i} = \frac{\begin{vmatrix} -\frac{\partial PW}{\partial A_i} & \frac{\partial PW}{\partial \log K} \\ -\frac{\partial G}{\partial A_i} & \frac{\partial G}{\partial \log K} \end{vmatrix}}{|J|}; \qquad \frac{\partial \log K^Q}{\partial A_i} = \frac{\begin{vmatrix} \frac{\partial PW}{\partial \log W_i} & -\frac{\partial PW}{\partial A_i} \\ \frac{\partial G}{\partial \log W_i} & -\frac{\partial G}{\partial A_i} \end{vmatrix}}{|J|}; \tag{32}$$

Here again all partial derivatives should be values at the equilibrium point, and need to be considered separately for Kaldorian underemployment and for maturity, because of the piecewise nature of the product wage schedule.

Following a conditional line of reasoning, let us suppose first that the generic equilibrium Q occurs during the Kaldorian underemployment phase; accordingly, the relevant expressions for the partial derivatives should be computed from equations 15 and 16. After some algebra (shown with more detail in Mathematical Appendix III.B) the above formulas reduce to

$$\frac{\partial \log W_i^Q}{\partial A_i} = -\frac{b\, L_i}{A_i\, \beta\, L_a} \frac{1}{|J^{KU}|}; \qquad \frac{\partial \log K^Q}{\partial A_i} = -\frac{1 - L_i\,(1\, -\, \gamma\, b)}{A_i\, \gamma\, \beta\, L_a} \frac{1}{|J^{KU}|}; \tag{33}$$

which imply, under the assumed parametrization, that the derivatives $\partial \log W_i^Q / \partial A_i$ and $\partial \log K^Q / \partial A_i$ take the opposite sign of $|J^{KU}|$.

To complete the conditional analysis, suppose instead that the equilibrium point Q belongs to the maturity interval; in such case, the relevant partial derivatives in expression 32 should be computed from equations 20 and 16. After some algebraic manipulation this operation obtains:

$$\frac{\partial \log W_i^Q}{\partial A_i} = -\frac{b\, L_i}{A_i\, \beta\, L_a} \frac{1}{|J^{MA}|}; \qquad \frac{\partial \log K^Q}{\partial A_i} = -\frac{1 - L_i\,(1\, -\, b)}{A_i\, \beta\, L_a} \frac{1}{|J^{MA}|}; \tag{34}$$

in which J^{MA} indicates the Jacobian corresponding to the maturity interval. Equation 34 implies that the derivatives $\partial \log W_i^Q / \partial A_i$ and $\partial \log K^Q / \partial A_i$ take the opposite sign of $|J^{MA}|$.

Like in the previous case, the correspondence principle ensures that the sign of $|J^{KU}|$ and $|J^{MA}|$ in 33 and 34 can be univocally determined, and it is hence possible to recap the comparative statics results in proposition III.

Proposition III *Parametric increases in industrial TFP reduce the basin of attraction of the locally stable equilibrium of pure subsistence, and move the stable equilibrium (if any) towards North-East, increasing the steady state value of capital and wages.*

Proposition III is illustrated graphically in figure 4, which considers the case in which there is also a stable equilibrium of full industrialization[56] (dashed schedules represent the equilibrium loci before the productivity increase). The economic explanation goes as follows, regardless of which phase the economy goes through. The increase in A_i raises *ceteris paribus*, the supply of manufactures, and stimulate the reallocation of labor towards industry. Given that agriculture displays decreasing returns

[56] Clearly had the situation been that of figure 2a, the only change would have been on the unstable equilibrium T.

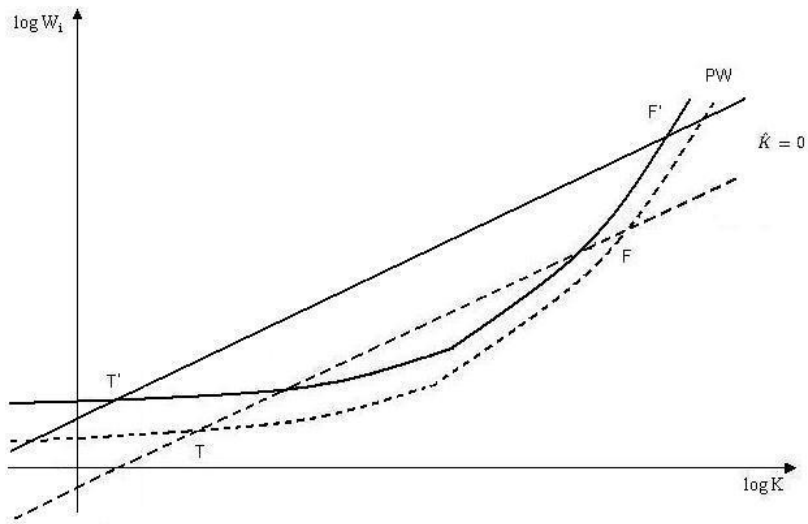

Figure 4: The effect of an increase in industrial TFP

to labor, the increase in L_i is bound to rise the agricultural wages, which in turn trigger an upwards adjustment of the nominal industrial wages. These factors explain the upwards move of the PW schedule. The raise in industrial productivity brings, however, a much larger gain to entrepreneurs, boosting their profits, and allowing a faster capital accumulation; this is reflected in the upwards shift of the stationary capital locus. Since the vertical movement of the $\hat{K} = 0$ locus outweighs that of the product wage schedule[57], the unstable low-development equilibrium will occur for a lower level of capital stock. Technical progress in industry directly boosts the profitability of entrepreneurs, so that a self-sustaining accumulation of capital becomes viable even for lower capital stocks. For exactly the same reasons, the equilibrium of full industrialization - if any - will always be pushed towards higher levels of capital stock by improvements in industrial TFP, regardless of the phase of the economy.

VI. *CONCLUSIONS*

In line with our main objective, we have combined in this two-sector macro-model several aspects emphasized by the neoclassical theory of growth and structural change, with other insights drawn from the more dated literature about dual economies and industrialization. Interestingly, the adoption of an efficiency wage mechanism in the urban labor market (unlike in the rural one) and the presence of technological external economies in industry, are sufficient to rationalize a view of the agriculture-industry shift à la Kaldor, and to originate poverty traps. Of course, Kaldor's structure of causality pivots around the central role of effective demand, while we retain a Ricardian supply-driven framework, resembling in this respect Lewis's model of unlimited supply of labor. Nevertheless, the complex interactions between agriculture and industry, the importance of labor reallocation to the more dynamic sector, and the asymmetric working of the labor market represent common aspects that link the present work to Kaldor's "Strategic factors in economic development", and highlight the crucial role of industrialization and increasing returns in the process of development.

As concerns instead the long debate on the big push argument, the above analysis has shown how even moderate degrees of increasing returns in industry are sufficient to give rise to poverty traps, since their effect is reinforced by the elastic supply of labor for the more dynamic industrial sector.

[57]This can be verified by directly computing $\partial \log W_i / \partial A_i$ for the product wage schedule and for the stationary capital locus: the derivative in the latter case outweighs the correspondent derivative for PW.

The nature of the poverty trap in the present set-up is closely linked to the productive specialization of the economy, and suggests that there may be structural factors that prevent the autonomous expansion of the industrial base at low levels of development. While these results are encouraging, our model - likewise the majority of poverty trap models - surely "tends to be lacking in testable quantitative implications"[58]. Nevertheless, the mechanisms outlined here are consistent with recent empirical evidence on multiple growth regimes[59], and seem confirmed by the simulations presented in Graham and Temple (2005), which suggest that multiple equilibria may have a saying in explaining income dispersion across countries and are particularly suitable to characterize the poorest LDCs, those with the closest conditions to our theoretical framework: extremely capital-poor agricultural sector and widespread areas of subsistence agriculture.

Besides, in the third section we have shown the impact of technical progress, identified here with parametric increases in the sectoral TFPs. Interestingly, while technical progress in industry has a undisputable positive effect (not just in terms of welfare but also of favoring the specialization in the more dynamic sector), situations of Dutch disease follow from increases in the agricultural TFP, locking in the specialization of the country in primary commodities or in any case penalizing the industrial sector, and lowering the steady state value of wages and capital stock.

Mathematical Appendix

I. THE KALDORIAN UNDEREMPLOYMENT PHASE

Combining the agricultural production function (equation 2) with the equations determining the rural wage rate and the rents (respectively 4 and 5), yields

$$\frac{1}{1-b} W_a L_a = P_a^* A_a L_a^{1-b};$$

on the other hand, the determination of the efficiency wage implies

$$W_a = \left(\frac{1-d}{\omega}\right)^{\frac{1}{d\gamma}} W_i^{\frac{1}{\gamma}} (\underline{P}^*)^{-\frac{1-\gamma}{\gamma}}. \tag{35}$$

Combining the two above equations, and making use of the labor market clearing (equation 10), obtains

$$\frac{1}{1-b}\left(\frac{1-d}{\omega}\right)^{\frac{1}{d\gamma}} \frac{1}{A_a P_a^* (\underline{P}^*)^{\frac{1-\gamma}{\gamma}}} W_i^{\frac{1}{\gamma}} (1-L_i)^b = 0; \tag{36}$$

from which log-differentiation yields the value of the industrial labor supply elasticity as mentioned in equation 14.

Furthermore, substituting in equation 36 L_i with its short-run value from equation 8, yields

$$\frac{1}{1-b}\left(\frac{1-d}{\omega}\right)^{\frac{1}{d\gamma}} \frac{1}{A_a P_a^* (\underline{P}^*)^{\frac{1-\gamma}{\gamma}}} W_i^{\frac{1}{\gamma}} \left[1 - A_i^{\frac{1}{\beta}} \Phi \exp\left(\frac{\mu+\beta}{\beta}\log K - \frac{1}{\beta}\log W_i\right)\right]^b = 0;$$

which expressed in logarithmic terms is precisely the product wage schedule mentioned in the text (equation 15).

[58] The quotation is taken from Azariadis and Stachurski (2005), recognizing a limit which is common to most models of poverty trap.

[59] See Durlauf and Johnson (1995); Durlauf, Kourtellos and Minkin (2001) and Durlauf, Johnson and Temple (2005).

Finally, concerning the evolution of the wage gap, rewrite the ratio between agricultural and industrial wage[60] making use of equation 35 to obtain

$$\frac{W_a}{W_i} = \left(\frac{1 - d}{\omega}\right)^{\frac{1}{d\,\gamma}} W_i^{\frac{1}{\gamma}} \, (\underline{P}^*)^{-\frac{1-\gamma}{\gamma}} \, W_i^{\frac{1-\gamma}{\gamma}};$$

which by construction is lower than 1 during the Kaldorian underemployment phase. Taking logs, and differentiating with respect to $\log K$ obtains

$$\frac{\partial \log \frac{W_a}{W_i}}{\partial \log K} = \frac{1 - \gamma}{\gamma} \frac{\partial \log W_i}{\partial \log K} > 0;$$

which is greater than zero, given the result of equation 17.

Since this derivative is strictly positive for the parametrization assumed above, the wage ratio tends to grow along with increases in the capital stock, ultimately reaching one when the system enters the maturity phase and wage gap disappear. To see this, note that the logarithm is a monotonically increasing transformation of the wage ratio and of the capital stock, hence the sign of the log-derivative $\partial \log \frac{W_a}{W_i} / \partial \log K$ equals the sign of the simple derivative of the wage ratio to capital stock.

II. THE MATURITY PHASE

During maturity the prevalence of an uniform wage rate implies, combining equations 11, 4 and 10,

$$W_a = W_i = (1 - b)\, A_a\, (1 - L_i)^{-b}\, P_a^*; \tag{37}$$

then, taking logs and totally differentiating obtains the elasticity of industrial labor supply faced by entrepreneurs, as expressed in equation 19 of the text.

Further, to obtain the equation of the product wage schedule during the maturity phase, replace the industrial labor demand in equation 37 with its value from equation 8, and recall that labor efficiency in the maturity phase will still be given by E^*. This yields

$$W_i = A_a\, (1 - b) \left[1 - A_i^{\frac{1}{\beta}}\, \Phi \, \exp\left(\frac{\mu + \beta}{\beta} \log K - \frac{1}{\beta} \log W_i\right)\right]^{-b} P_a^*; \tag{38}$$

Taking logs, obtains from this expression the product wage schedule as given in the text (equation 20).

III. COMPARATIVE STATICS: THE EFFECT OF TECHNICAL CHANGE

A. PROOF OF PROPOSITION II

From the previous analysis it should be clear that in system 26 the relevant equations during Kaldorian underemployment are actually 15 for PW and 16 in place of G. Accordingly, the following magnitudes are of interest for comparative statics in the Kaldorian underemployment phase:

$$J^{KU} \equiv \begin{pmatrix} \frac{\partial PW}{\partial \log W_i} & \frac{\partial PW}{\partial \log K} \\ \frac{\partial G}{\partial \log W_i} & \frac{\partial G}{\partial \log K} \end{pmatrix} = \begin{pmatrix} \frac{\beta\, L_a + \gamma b L_i}{\gamma\, \beta\, L_a} & -\frac{(\mu+\beta) b L_i}{\beta\, L_a} \\ -\frac{1-\beta}{\beta} & \frac{\mu}{\beta} \end{pmatrix};$$

and

$$\frac{\partial PW}{\partial A_a} = -\frac{1}{A_a}; \qquad \frac{\partial G}{\partial A_a} = 0.$$

[60]Note that the absolute wage gap is tied to the ratio by the following relation $W_i - W_a = (1 - W_a/W_i)\, W_i$.

Replacing the partial derivatives of equation 28 with the corresponding values as determined here, obtains after some manipulation 29.

As concerns the sign of $|J^{KU}|$, its direct calculation shows after some algebra that

$$|J^{KU}| > 0 \iff \epsilon_{ku}^{LS} > \frac{1-\beta-\mu}{\mu};$$

which basically verifies the correspondence principle between statics and dynamics.

During maturity, instead, the relevant equations for system 26 are number 20 (for PW) and 16 (for G); accordingly we have the following magnitudes

$$J^{MA} \equiv \begin{pmatrix} \frac{\partial PW}{\partial \log W_i} & \frac{\partial PW}{\partial \log K} \\ \frac{\partial G}{\partial \log W_i} & \frac{\partial G}{\partial \log K} \end{pmatrix} = \begin{pmatrix} \frac{\beta\, L_a + bL_i}{\beta\, L_a} & -\frac{(\mu+\beta)bL_i}{\beta\, L_a} \\ -\frac{1-\beta}{\beta} & \frac{\mu}{\beta} \end{pmatrix};$$

$$\frac{\partial PW}{\partial A_a} = -\frac{1}{A_a}; \qquad \frac{\partial G}{\partial A_a} = 0.$$

Replacing the partial derivatives in equation 28 with the corresponding values determined here for the maturity phase, directly obtains 30.

On the other hand, the direct calculation of $|J^{MA}|$ verifies the correspondence principle, establishing precisely that

$$|J^{MA}| > 0 \iff \epsilon_{ma}^{LS} > \frac{1-\beta-\mu}{\mu};$$

and with this last condition the sign of the derivatives $\partial \log W_i^Q / \partial A_a$ and $\partial \log K^Q / \partial A_a$ can be univocally determined, as done in the text.

B. PROOF OF PROPOSITION III

Starting with Kaldorian underemployment, the relevant equations for system 31 are 15 (for PW), and 16 (for G). Hence, in addition to the matrix J^{KU} defined above, the magnitudes of interest for the comparative statics regarding A_i in the Kaldorian underemployment phase are:

$$\frac{\partial PW}{\partial A_i} = -\frac{bL_i}{\beta L_a}\frac{1}{A_i}; \qquad \frac{\partial G}{\partial A_i} = \frac{1}{\beta}\frac{1}{A_i}.$$

Replacing these values for the corresponding partial derivatives in equation 32 directly obtains 33. Recalling, finally, the condition for a positive determinant of J^{KU}, yields the comparative statics result mentioned in the text.

As concerns the maturity phase, instead, the relevant Jacobian is J^{MA} defined above, and from equations 20 (for PW) and 16 (for G), it is possible to compute the magnitudes

$$\frac{\partial PW}{\partial A_i} = -\frac{bL_i}{\beta L_a}\frac{1}{A_i}; \qquad \frac{\partial G}{\partial A_i} = \frac{1}{\beta}\frac{1}{A_i}.$$

Substituting these expressions in equation 32 yields equation 34.

Finally, using the correspondence principle to determine whether $|J^{MA}|$ is positive or negative, one can univocally establish the sign of the derivatives $\partial \log W_i^Q / \partial A_i$ and $\partial \log K^Q / \partial A_i$.

References

[1] Acemoglu, D. Training and innovation in an imperfect labor market. *Review of Economic Studies 64* (1997), 445–464.

[2] Aghion, P., and Bolton, P. A theory of trickle-down growth and development. *Review of Economic Studies 64* (1997), 151–172.

[3] Akerlof, G. Labor contracts as partial gift exchange. *Quarterly Journal of Economics 97* (1982), 543–569.

[4] Akerlof, G., and Yellen, J. *Efficiency wage models of the labor market.* Cambridge University Press, 1986.

[5] Arida, P., and Taylor, L. *Long-run Income Distribution and Growth.* (in "Handbook of development economics") Chenery and Srinivasan (editors) Elsevier Science Publishers, 1988.

[6] Arrow, K. The economic implications of learning by doing. *Review of Economic Studies 29* (1962), 155–173.

[7] Azariadis, C. *The theory of poverty traps: what have we learned?* (in "Poverty Traps") Bowles, Durlauf and Hoff (editors) Princeton University Press, 2005.

[8] Azariadis, C., and Drazen, A. Threshold externalities in economic development. *The Quarterly Journal of Economics 105* (1990), 501–526.

[9] Azariadis, C., and Stachurski, J. *Poverty traps.* (in "Handbook of economic growth") Aghion and Durlauf (editors) Elsevier Science Publishers, 2005.

[10] Banerjee, A., and Newman, A. Occupational choice and the process of development. *Journal of Political Economy 101* (1993), 274–298.

[11] Banerjee, A., and Newman, A. Information, the dual economy and development. *Review of Economic Studies 65* (1998), 631–653.

[12] Barro, R. Economic growth in a cross section of countries. *The Quarterly Journal of Economics 106* (1991), 407–443.

[13] Barro, R., and Sala-i Martin, X. *Economics growth.* McGraw-Hill, 1995.

[14] Basu, K. *The Less Developed Economy A critique of contemporary theory.* Basil Blackwell, 1984.

[15] Basu, K. *Analytical Development Economics: The Less Developed Economy Revisited.* MIT press, 1997.

[16] Baumol, W. J. Macroeconomics of unbalanced growth: the anatomy of the urban crisis. *American Economic Review 57* (1967), 185–196.

[17] Ben David, D. Convergence clubs and subsistence economies. *Journal of Development Economics 55* (1998), 155–171.

[18] Bhaduri, A. *Alternative development strategies and the rural sector.* (in "Economic crisis and the Third World agriculture") Singh and Tabatabai (editors) Cambridge University Press, 1993.

[19] Bhaduri, A. *Structural change and economic development: on the relative roles of effective demand and the price mechanism in a dual economy.* (in "Rethinking development economics") Ha Joon Chang (editor) Anthem Press, 2003.

[20] Bhaduri, A., and Marglin, S. Unemployment and the real wage: the economic basis for contesting political ideologies. *Cambridge Journal of Economics 14* (1990), 375–393.

[21] Bhaduri, A., and Skarstein, R. Effective demand and the terms of trade in a dual economy: a kaldorian perspective. *Cambridge Journal of Economics 27* (2003), 583–595.

[22] Bianchi, M. Testing for convergence: evidence from non-parametric multimodality tests. *Journal of Applied Econometrics 12* (1997), 393–409.

[23] Bowen, H., Hollander, A., and Viaene, J. *Applied international trade analysis.* University of Michigan Press, 1998.

[24] Bowles, S. The production process in a competitive economy: Walrasian, neo-hobbesian and marxian models. *American Economic Review 75* (1985), 16–36.

[25] Chenery, H., and Syrquin, M. *Patterns of development, 1950-1970.* Oxford University Press, 1975.

[26] Cimoli, M., and Rovira, S. Elites and structural inertia in latin america: an introductory note on the political economy of development. *Journal of Economic Issues 42 (2)* (2008), 327–348.

[27] Desdoigt, A. Patterns of economic development and the formation of clubs. *Journal of economic growth 4* (1999), 305–330.

[28] Dixit, A. *Models of dual economies.* (in "Models of economic growth") Mirlees and Stern (editors) Macmillan, 1989.

[29] Durlauf, S., and Johnson, P. Multiple regimes and cross-country growth behavior. *Journal of Applied Econometrics 10* (1995), 365–384.

[30] Durlauf, S., Johnson, P., and Temple, J. *Growth econometrics.* (in "Handbook of economic growth") Aghion and Durlauf (editors) Elsevier Science Publishers, 2005.

[31] Durlauf, S., Kourtellos, A., and Minkin, A. The local Solow growth model. *European Economic Review 45* (2001), 928–940.

[32] Dutta, J. The wage good constraint on a developing economy: Theory and evidence. *Journal of Development Economics 28* (1988), 341–363.

[33] Easterly, W. Reliving in the 50s: the big push, poverty traps, and takeoffs in economic development. *mimeo* (2006).

[34] Fei, J., and Ranis, G. A theory of economic development. *American Economic Review 51* (1961), 533–565.

[35] Fleming, M. External economies and the doctrine of balanced growth. *Economic Journal 65* (1955), 241–256.

[36] Galor, O. Convergence? inferences from theoretical models. *The Economic Journal 106* (1996), 1056–1069.

[37] Galor, O., and Zeira, J. Income distribution and macroeconomics. *Review of Economic Studies 60* (1993), 35–52.

[38] Gandolfo, G. *Economic dynamics.* Springer, 1997.

[39] Graham, B., and Temple, J. Rich nations, poor nations: how much can multiple equilibra explain? *mimeo* (2005).

[40] Graham, F. Some aspects of protection further considered. *Quarterly Journal of Economics 37* (1923), 199–227.

[41] Hansen, G., and Prescott, E. Malthus to Solow. *American Economy Review 92* (2002), 1205–1217.

[42] Harris, J. R., and Todaro, M. P. Migration, unemployment and development: a two sector analysis. *American Economic Review 60* (1970), 126–142.

[43] Hausmann, R., Pritchett, L., and Rodrik, D. Growth acceleration. *mimeo* (2005).

[44] Hoff, K. Beyond rosenstein-rodan: the modern theory of coordination problems in development. *Proceedings of the Annual World Bank Conference (Supplement to the World Bank Economic Review)* (2000).

[45] Jorgenson, D. The development of a dual economy. *Economic Journal 71* (1961), 309–334.

[46] Kaldor, N. *Strategic Factors in Economic Development.* Ithaca: Cornell University, 1967.

[47] Kaldor, N. Productivity and growth in manufacturing industry: a reply. *Economica 35* (1968), 385–391.

[48] Kraay, A., and Raddatz, C. Poverty traps, aid, and growh. *Journal of Development Economics 82* (2007), 315–347.

[49] Kremer, M. The o-ring theory of economic development. *Quarterly Journal of Economics 108* (1993), 551–575.

[50] Krugman, P. The narrow moving band, the dutch disease, and the competitive consequences of mrs. thatcher. *Journal of Development Economics 27* (1987), 41–55.

[51] Krugman, P. Toward a counter-counterrevolution in development theory. *Proceedings of the Annual World Bank Conference 1992 (World Bank Economic Review)* (1992).

[52] Kuznets, S. *Modern economic growth.* Yale University Press, 1966.

[53] Kuznets, S. S. Economic growth and income inequality. *American Economic Review 45* (1955), 1–28.

[54] Leibenstein, H. *Economic backwardness and economic growths.* John Wiley & Sons, 1957.

[55] Lewis, A. Economic development with unlimited supplies of labor. *Manchester School of Economics and Social Studies 28* (1954), 139–191.

[56] Lewis, A. Unlimited labor: further notes. *Manchester School of Economics and Social Studies 32* (1958), 1–32.

[57] Lucas, R. E. J. On the mechanics of economic development. *Journal of Monetary Economics 22* (1988), 3–42.

[58] Lucas, R. E. J. Making a miracle. *Econometrica 61* (1993), 251–272.

[59] Malthus, T. *Principles of Political Economy: Considered with a View to Their Practical Application, 2nd Ed.* New York: Augustus M. Kelley, 1836.

[60] Mankiw, G., Romer, D., and Weil, D. A contribution to the empirics of economic growth. *The Quarterly Journal of Economics 107* (1992), 407–437.

[61] Marshall, A. *Principles of economics.* Macmillan, 1920.

[62] Matsuyama, K. Agricultural productivity, comparative advantage, and economic growth. *Journal of Economic Theory 58* (1992), 317–334.

[63] Mazumdar, D. The marginal productivity theory of wages and disguised unemployment. *Review of Economic Studies 26* (1959), 190–197.

[64] Murphy, K., Shleifer, A., and Vishny, R. Income distribution, market size and industrialization. *Quarterly Journal of Economics 104* (1989), 537–564.

[65] Murphy, K., Shleifer, A., and Vishny, R. Industrialization and the big push. *Journal of Political Economy 97* (1989), 1003–1026.

[66] Murphy, K., Shleifer, A., and Vishny, R. Why is rent-seeking so costly to growth? *American Economic Review 83* (1993), 409–414.

[67] Nelson, R. A theory of the low level equilibrium trap in underdeveloped economies. *American Economic Review 46* (1956), 894–908.

[68] Nurkse, R. *Problems of Capital Formation in Underdeveloped Countries.* Oxford University Press, 1953.

[69] Pasinetti, L. *Structural Economic Dynamics.* Cambridge University Press, 1993.

[70] PREOBRAZHENSKY, E. *The new economics.* Oxford University Press, 1965.

[71] QUAH, D. Galton's fallacy and tests of the convergence hypothesis. *The Scandinavian Journal of Economics 95* (1993), 427–443.

[72] QUAH, D. Twin peaks: growth and convergence in models of distribution dynamics. *The Economic Journal 106* (1996), 1045–1055.

[73] RANIS, G. *Analytics of Development: Dualism.* (in "Handbook of development economics") Chenery and Srinivasan (editors) Elsevier Science Publishers, 1988.

[74] RAY, D. *Development economics.* Princeton University Press, 1998.

[75] ROS, J. *Development theory and the economics of growth.* The University of Michigan Press, 2000.

[76] ROS, J., AND SKOTT, P. The big push in an open economy with non-tradable inputs. *Journal of Post Keynesian Economics 20* (1997), 149–162.

[77] ROSENSTEIN-RODAN, P. Problems of industrialization in Eastern and South-Eastern Europe. *Economic Journal 53* (1943), 202–211.

[78] ROSENZWEIG, M. R. *Labor Markets in Low Income Countries.* (in "Handbook of development economics") Chenery and Srinivasan (editors) Elsevier Science Publishers, 1988.

[79] SACHS, J. *The end of poverty: how we can make it happen in our lifetime.* Penguin Books, 2005.

[80] SACHS, J., MCARTHUR, J., , SCHMIDT-TRAUB, G., KRUK, M., BAHADUR, C., FAYE, M., AND MCCORD, G. Ending Africa's poverty trap. *Brookings Papers on Economic Activity, 1:Journal of Development Economics 1* (2004), 117–231.

[81] SALA-I MARTIN, X. The classical approach to convergence analysis. *The Economic Journal 106* (1996), 1019–1036.

[82] SALA-I MARTIN, X. I just ran two million regressions. *The American Economic Review 87* (1997), 178–183.

[83] SCITOVSKY, T. Two concepts of external economy. *Journal of Political Economy 62* (1954), 143–151.

[84] SEN, A. Peasants and dualism with or without surplus labor. *Journal of Political Economy 74* (1966), 425–450.

[85] SMITH, A. *Early draft of part of the Wealth of Nations.* (in "Lectures of Jurisprudence") Meek, Raphael and Stein (editors) Claredon Press, 1978.

[86] SOLOW, R. M. A contribution to the theory of economic growth. *Quarterly Journal of Economics 70* (1956), 65–94.

[87] SOLOW, R. M. Technical change and the aggregate production function. *The Review of Economics and Statistics 39* (1957), 312–320.

[88] SOLOW, R. M. Perspectives on growth theory. *Journal of Economic Perspectives 8* (1994), 45–54.

[89] STIGLITZ, J. The Efficiency Wage hypothesis, surplus labor, and the distribution of labor in LDCs. *Oxford Economic Papers 28* (1976), 185–207.

[90] STIGLITZ, J. Comment on toward a counter-counterrevolution in development theory. *Proceedings of the Annual World Bank Conference 1992 (World Bank Economic Review)* (1992).

[91] STIGLITZ, J. E. *Economic Organization.* (in "Handbook of development economics") Chenery and Srinivasan (editors) Elsevier Science Publishers, 1988.

[92] STOKEY, N. Learning by doing and the introduction of new goods. *Journal of Political Economy 96* (1988), 701–717.

[93] SYRQUIN, M. *Sector proportions and Economic Development: the evidence since 1950.* (in "The balance between agriculture and industry in economic development") Panchamukhi and Williamson(editors) Macmillan, 1989.

[94] TAYLOR, L. *Structuralist Macroeconomics.* New York: Basic Books, 1983.

[95] TAYLOR, L. *Theories of Sectoral Balances.* (in "The balance between agriculture and industry in economic development") Panchamukhi and Williamson(editors) Macmillan, 1989.

[96] TODARO, M. P. A model of labor migration and urban unemployment in Less Developed Countries. *American Economic Review 59* (1969), 138–148.

[97] TORVIK, R. Learning by doing and the dutch disease. *European Economic Review 45* (2001), 285–306.

[98] VALENSISI, G. *Dualism and the big push.* (in "Geography, Structural Change and Economic Development: Theory and Empirics") Salvadori, Tamberi and Commendatore (editors); Edward Elgar Publishing, forthcoming.

[99] VERDOORN, P. J. Fattori che regolano lo sviluppo della produttività del lavoro. *L'industria 1* (1949), 45–53.

[100] WILLIAMSON, J. G. *Migration and Urbanization.* (in "Handbook of development economics") Chenery and Srinivasan (editors) Elsevier Science Publishers, 1988.

[101] YOUNG, A. Increasing returns and economic progress. *Economic Journal 38* (1928), 527–542.