Resource Wealth, Innovation and Growth in the Global Economy*

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January 21, 2010

Abstract

We analyze the relative growth performance of open economies in a two-country model where different endowments of labor and a natural resource generate asymmetric trade. A resource-rich economy trades resource-based intermediates for final manufacturing goods produced by a resource-poor economy. Productivity growth in both countries is driven by endogenous innovations. The effects of a sudden increase in the resource endowment depend crucially on the elasticity of substitution between resources and labor in intermediates' production. Under substitution (complementarity), the resource boom generates higher (lower) resource income, lower (higher) employment in the resource-intensive sector, higher (lower) knowledge creation and faster (slower) growth in the resource-rich economy. The resource-poor economy adjusts to the shock by raising (reducing) the relative wage, and experiences a positive (negative) growth effect that is exclusively due to trade.

Keywords: Endogenous Growth, Endogenous Technological Change, Natural Resources, International Trade.
JEL Classification Numbers: E10, F43, L16, O31, O40

1 Introduction

The distribution of primary resources across countries is an important determinant of trade patterns. Virtually every economy endowed with natural resources that can be processed into essential factors of production exports such resource-based commodities and imports manufacturing goods from resource-poor economies. This asymmetric trade structure stems from different endowments and creates interesting interdependencies: while resource-poor economies specialize in manufacturing by force of nature, they gain from trading non-primary goods demanded by resource-rich countries specialized in primary production. Lederman and Maloney (2007) document that the links between trade structure and economic performance are empirically robust: trade variables, especially natural resource abundance and export concentration, are important determinants of economic growth.

*We thank Corrado Di Maria, Michael Moore, Rick Van der Ploeg, John Seater and seminar participants at Duke University (TDM), University of Oxford (OxCarre) and Queen's University of Belfast for comments and suggestions. Simone Valente gratefully acknowledges financial support by the Swiss National Science Foundation (grant IZK0Z1-125589).

Although typical in reality, asymmetric trade structures do not play a prominent role in the theoretical literature on international trade and economic growth. The benchmark framework of two-country models (e.g., Grossman and Helpman 1991) concentrates on the role of endogenous innovation in generating convergence across trading economies that exhibit productivity differences (Feenstra, 1996; Barro and Sala-i-Martin 1997; Acemoglu and Zilibotti, 2001) and neglects the role of natural resources in driving trade specialization. Natural resource abundance is prominent only in the parallel literature on "Dutch Disease" phenomena (Corden, 1984) and/or the "Curse of Natural Resources" (Sachs and Warner, 1995) — situations in which greater resource wealth produces negative effects on productivity growth — but these contributions focus on resource-rich countries characterized as small open economies where asymmetric trade with resource-poor countries plays at best a minor role.

In this paper, we develop a two-country model of R&D-based growth featuring both inter-industry and intra-industry trade. The resource-rich economy, that we call Home, exports resource-based intermediates and differentiated final manufacturing goods and imports only differentiated final goods from the resource-poor economy, that we call Foreign. Both economies develop innovations that generate endogenous productivity growth. In this framework, we analyze the effects of *resource booms* — sudden increases in Home's natural resource endowment due to unexpected discoveries — on expenditure levels, innovation rates, productivity growth, the allocation of labor across sectors, and welfare in both economies.

Our analysis differs from the Dutch-Disease literature in both aims and means. At the theoretical level, our framework (i) distinguishes between physical resource endowments and flows of resource income, (ii) assumes that the resource-based sector is vertically related to both domestic and foreign manufacturing sectors, and (iii) is a fully general equilibrium model where international prices are endogenously determined by the world market equilibrium. At the conceptual level, we do not attempt to explain Dutch-Disease phenomena. In our model greater resource abundance may yield slower or faster growth, but our aim is to characterize the transmission channels between shocks to resource endowments, income levels, economic growth and welfare in the presence of asymmetric trade.

This research question is empirically important because the correlation between resource abundance — literally interpreted as the size of the *physical endowment* of natural resources — and economic performance is still unclear. When Sachs and Warner (1995) first documented a negative correlation and developed the so-called resource curse hypothesis, they identified resource abundance with the ratio of resource exports to gross domestic product. This is an imperfect proxy for physical abundance and is more a measure of specialization. Recent empirical work by Brunnschweiler and Bulte (2008) shows that measuring resource abundance with stock-based indices — which are much better proxies of the size of endowments — reveals that the data strongly reject the resource curse hypothesis since resource abundance is positively correlated with growth and income levels.[1]

Our characterization of the transmission channels between resource booms and economic growth builds on two considerations. First, in reality natural endowments are not directly

[1]The resource curse hypothesis is also challenged by Lederman and Maloney (2007) albeit in a different way. Since the Sachs and Warner's (1995) measure of resource abundance incorporates effects due to specialization and trade dependence, Lederman and Maloney (2007) disentangle these effects by keeping the original measure of abundance while controlling for export concentration in panel-based estimates. Their results reject the original resource curse hypothesis: growth rates are found to be positively correlated to resource abundance and negatively correlated to export concentration.

consumed but exploited by resource-processing firms that sell intermediate inputs to manufacturing sectors producing final goods. Since the international price of resource-intensive goods is endogenous, the elasticity of global demand — the sum of domestic and foreign demands for the processed resource — is an important determinant of the transmission of endowments shocks. Since the primary sectors of resource-rich countries are vertically related to both domestic and foreign final sectors, the elasticity of the demand for intermediates reflects the characteristics of the technology employed by final producers. Second, international trade implies that real income growth in each country depends on the productivity growth rates in the other economies. In a two-country framework endowment shocks in Home may affect domestic productivity growth and leave Foreign productivity growth unchanged, but real income growth in Foreign can nonetheless be affected by the shock through trade since Home's productivity matters for Foreign's capacity to import. We address these points by studying the effects of resource booms in a model where resource-based intermediates may be either complements or substitutes with other inputs in the final sector and intra-industry trade in final goods induces feedback growth effects between the two countries.

We formalize the vertical structure of intermediate and final sectors following the closed-economy model of Peretto (2008), which we extend to include a second country and asymmetric trade. Both economies develop horizontal *and* vertical innovations: given total market size, the rate of horizontal innovation determines the size of firms, which in turn drives the rate of vertical innovation and thereby long-run growth. An important property of the model is that steady-state growth rates are independent of endowments since the market structure absorbs scale effects in the long run. One rationale for employing this framework is empirical plausibility: in models displaying scale effects, the growth rate depends on initial endowments and a resource boom has permanent growth effects. In our model, instead, resource booms affect total factor productivity (TFP) growth during the transition and yield permanent effects on the allocation of labor and on the number of firms operating in the resource-rich economy. Another rationale for our framework is tractability: we solve the model in closed form and thus have a transparent characterization of growth and welfare.

Our findings concern three main issues: the nature of the transmission mechanism, the direction of income and growth effects, and the transitional dynamics implied by the reallocation of labor across sectors induced by a resource boom. We show that a sudden rise in Home's resource endowment induces a change in the demand for manufacturing goods produced by Foreign, which triggers a change in the relative wage but leaves Foreign's TFP growth unaffected. In Home, instead, labor moves across the primary and manufacturing, with permanent effects on the equilibrium number of manufacturing firms and transitional effects on TFP growth. The balanced trade condition then implies that the variation in TFP growth experienced by Home shows up in Foreign's import price index. In other words, there is transmission due to trade of the dynamic effects of the resource boom to the resource-poor country.

With respect to the direction of income and growth effects, our analysis emphasizes the role of global resource demand. We show that the sign of the growth effect depends on whether labor and the raw resource are complements or substitutes in the production of resource-based intermediates. If they are substitutes, a resource boom raises Home's resource income and its overall expenditure on manufacturing goods. In Foreign, the wage increases due to Home's higher demand for manufacturing goods. In Home, labor moves into manufacturing and there is a permanent positive effect on the equilibrium number of firms. During the transition, the process of entry raises Home's TFP growth. This positive growth effect in Home is then transmitted to Foreign through trade since households in each country consume both domestic and imported goods. If labor and the raw resource are complements, instead, the same mechanism works in the opposite direction: a positive shock to Home's resource endowment lowers resource income and thereby Home's demand for Foreign's manufacturing goods. Consequently, Foreign's wage falls while Home's number of firms converges to a lower steady-state level and the associated transitional growth effects are negative for both countries. The intuition behind these results is that the elasticity of substitution in the intermediate sector determines the reaction of Home's resource income to an increase in the resource endowment. Substitutability implies elastic demand for the raw resource, which means that a positive shock to the resource supply requires a mild reduction in the resource price so that the net effect on resource income is positive. Complementarity, instead, implies inelastic demand, which means that the resource boom causes a fall of the resource price so drastic that it more than offsets the larger quantity sold with the result that resource income falls.

The transitional dynamics of sectoral labor employment in the Home economy are driven by the coexistence of vertical and horizontal innovations. The amount of labor employed in the manufacturing sector, in particular, is subject to two effects pushing in the same direction. When the resource boom yields higher (lower) resource income, the first effect is a sudden increase (reduction) in labor employed in final production due to the upward (downward) jump in expenditure on manufacturing goods. Subsequently, the process of net entry (net exit) of manufacturing firms yields further increases (decreases) of employment in the production of final goods in Home. Hence, the crowding-in (crowding-out) effects generated by resource booms are self-reinforcing due to the two, interdependent dimensions of innovation.

The plan of the paper is as follows. Section 2 describes the model assumptions. Section 3 characterizes the world competitive equilibrium and derives the main results. Section 4 discusses the connections with previous literature, and section 5 concludes.

2 The Model

There are two countries and two factors of production: Home, denoted H, has labor and a natural resource; Foreign, denoted F, has only labor. Home uses the natural resource and labor to produce a homogeneous, resource-based intermediate input. Both countries combine the resource-based input with labor to produce differentiated manufacturing goods. Consequently, Home exports both resource-based and manufacturing goods while Foreign exports only manufacturing goods. Labor is homogeneous and moves freely across sectors within each country; for simplicity, it does not move across countries. Each economy develops both horizontal and vertical innovations.

2.1 Households

Each country is populated by a representative household with L^J members, where $J = H, F$ is the country index. Each household member supplies inelastically one unit of labor, and population is assumed constant for simplicity. Individual utility is a weighted average of consumption of home and foreign goods.

Preferences. Each country produces N^J varieties of consumption goods. X_i^J, $i \in \left[0, N^J\right]$

is the quantity of good i produced in country J. Since this quantity satisfies both domestic and foreign consumption, we can write $X_i^H = X_i^{Hh} + X_i^{Hf}$, where X_i^{Hh} is the quantity consumed in H and X_i^{Hf} is the quantity exported to F. Symmetrically, manufacturing production in Foreign equals $X_i^F = X_i^{Ff} + X_i^{Fh}$, where X_i^{Ff} is the quantity consumed in F and X_i^{Fh} is the quantity exported to H. Instantaneous utility in each country is:

$$\log u^H = \xi \log \left[\int_0^{N^H} \left(\frac{X_i^{Hh}}{L^H} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} + (1-\xi) \log \left[\int_0^{N^F} \left(\frac{X_i^{Fh}}{L^H} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} ; \quad (1)$$

$$\log u^F = (1-\xi) \log \left[\int_0^{N^H} \left(\frac{X_i^{Hf}}{L^F} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} + \xi \log \left[\int_0^{N^F} \left(\frac{X_i^{Ff}}{L^F} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} , \quad (2)$$

where $\epsilon > 1$ is the elasticity of substitution across goods and $\xi \in [1/2, 1)$ is domestic bias, i.e., the weight that each consumer assigns to utility from consuming goods produced in her country of residence. For $\xi = 1/2$, the model reduces to a standard one of symmetric preferences across countries. We rule out $\xi = 1$ because it yields no trade. We assume integrated world markets and abstract from trade frictions (e.g., tariffs, transport costs) so that the law of one price holds and P_i^J, the price of good i produced in country J, applies in both Home and Foreign.

Utility maximization. The household maximizes

$$U_0^J = \int_0^{\infty} e^{-\rho t} \log u^J(t)\, dt, \quad (3)$$

where $\rho > 0$ is the time-preference rate. The instantaneous expenditure constraints are:

$$E^H = \int_0^{N^H} P_i^H X_i^{Hh} di + \int_0^{N^F} P_i^F X_i^{Fh} di; \quad (4)$$

$$E^F = \int_0^{N^H} P_i^H X_i^{Hf} di + \int_0^{N^F} P_i^F X_i^{Ff} di, \quad (5)$$

where E^J is the expenditure of residents of country J on consumption goods. The wealth constraints are:

$$\dot{A}^H = r^H A^H + W^H L^H + p\Omega + \Pi_M - E^H; \quad (6)$$

$$\dot{A}^F = r^F A^F + W^F L^F - E^F, \quad (7)$$

where A^J is assets, r^J the rate of return to assets, and W^J the wage rate. Since the resource endowment and the resource-processing sector are located in Home, constraint (6) also includes dividend income from resource-processing firms, Π_M, and resource income $p\Omega$, where Ω represents the resource endowment and p the market price of the resource.

2.2 Manufacturing sector

The manufacturing sector consists of firms producing differentiated consumption goods. In line with the growth literature, we have single-product firms so that N^J represents the mass of manufacturing firms operating in country J. Productivity growth stems from both vertical and horizontal innovations. Each manufacturing firm performs R&D to increase its own total factor productivity. At the same time, outside entrepreneurs perform R&D to develop new products and then set up firms to serve the market. This process of horizontal innovation increases the mass of firms N^J over time.

Production. Each firm in each country produces with the technology

$$X_i^J = \left(Z_i^J\right)^{\theta} \cdot \left[\left(M_i^J\right)^{\sigma}\left(L_{X_i}^J - \phi\right)^{1-\sigma}\right], \quad i \in \left[0, N^J\right], \tag{8}$$

where X_i^J is the quantity produced, $L_{X_i}^J$ is labor employed in production, $\phi > 0$ is a fixed labor cost, M_i^J is the quantity of the resource-based input, Z_i^J is the stock of firm-specific knowledge and $\theta \in (0, 1)$ is the elasticity of the firm's total factor productivity with respect to knowledge. Technology (8) exhibits constant returns to scale to rival inputs while the firm can raise its total factor productivity by raising its knowledge stock Z_i^J.

Vertical Innovation. Each firm's knowledge evolves according to

$$\dot{Z}_i^J = \alpha K^J \cdot L_{Z_i}^J, \tag{9}$$

where $\alpha > 0$ is a technological opportunity parameter,

$$K^J = \int_0^{N^J} \frac{1}{N^J} Z_i^J di. \tag{10}$$

is the stock of public knowledge in country J and $L_{Z_i}^J$ is labor employed in R&D activity.[2] For simplicity we set international knowledge spillovers at zero. Qualitatively, nothing of substance changes if they are positive (see, e.g., Peretto 2003).

Horizontal innovation (entry). The mass of manufacturing firms in each country evolves according to how much labor is devoted to developing new products and to starting up operations. For each entrant, denoted i without loss of generality, the labor requirement translates into a sunk cost that is proportional to the value of the production good. Formally, letting $L_{N_i}^J$ denote labor employed in start-up activity by each entrant, the entry cost equals $W^J L_{N_i}^J = \beta Y_i^J$, where $Y_i^J \equiv P_i^J X_i^J$ is the value of production of the new good when it enters the market and $\beta > 0$ is a parameter representing technological opportunity. This assumption captures the notion that entry requires more effort the larger the anticipated volume of production.[3] Entry creates value

$$V_i^J(t) = \int_t^{\infty} \Pi_{X_i}^J(v)\, e^{-\int_t^v \left[r^J(s)-\delta\right] ds} dv, \tag{11}$$

[2] Peretto and Smulders (2002) provide explicit micro-foundations for the knowledge aggregator (10).

[3] This assumption can be rationalized in several alternative ways and does not affect the generality of our results (see Peretto and Connolly 2007). In particular, we would obtain identical dynamics of the mass of firms by assuming, instead, a consolidated R&D sector producing blueprints with linear technology and making zero profits.

where $\Pi_{X_i}^J$ is the instantaneous profit flow, $r^J(v)$ is the instantaneous interest rate and $\delta > 0$ is the instantaneous death rate of firms.[4] Free entry requires

$$V_i^J = \beta Y_i^J = W^J L_{N_i}^J, \tag{12}$$

which says that the value of the new firm must equal the entry cost for each entrant.

2.3 Intermediate sector in Home

The intermediate input is produced by resource-processing firms in country H. This sector captures the key properties of the economic use of the natural resource and thus drives many of our results. In particular, resource-processing firms use a CES production technology that brings to the forefront the role of the degree of substitution/complementarity between labor and the natural resource in the production of the intermediate input. For simplicity, we assume that this sector is competitive.

Production. Let M^J be the quantity of resource-based input used in country J. The total output of the intermediate sector M is split between the quantities sold to domestic and to foreign final producers,

$$M = M^H + M^F = \int_0^{N^H} M_i^H di + \int_0^{N^F} M_i^F di,$$

where M_i^H is the amount of intermediate used by the i-th manufacturing firm in country J. The intermediate is produced by an indefinite number of identical competitive firms in Home under constant returns to scale. The technology of the intermediate sector is thus represented by

$$M = \left[\varsigma L_M^{\frac{\tau-1}{\tau}} + (1-\varsigma) R^{\frac{\tau-1}{\tau}} \right]^{\frac{\tau}{\tau-1}}, \tag{13}$$

where $\varsigma \in (0,1)$ is a weighting parameter and $\tau \geq 0$ is the elasticity of substitution: labor and the natural resource are complements when $\tau < 1$ and substitutes when $\tau > 1$. As $\tau \to 1$, technology (13) reduces to the Cobb-Douglas form $L_M^{\varsigma} R^{1-\varsigma}$. Total profits of the intermediate sector equal $\Pi_M = P_M M - W^H L_M - pR$, where P_M is the price of the resource-based good and p the price of the natural resource.

Market-clearing Equilibrium. The link between resource use, R, and the resource endowment, Ω, can take various forms. To keep the analysis simple, we assign full and well-defined property rights over the endowment to the household and assume full utilization at each point in time of a non-depletable resource stock. Hence, in equilibrium $R = \Omega$ and $\Pi_M = 0$ hold. The results do not change if R is a constant fraction of Ω, which would be the case if, e.g., the resource were renewable and the extraction rule to keep a constant flow of resource use. Since our aim is to study the effects of resource booms, represented by sudden increases in Ω, the assumption of constant resource use is not particularly restrictive: even in the case of exhaustible resources, a sudden jump in the resource stock would imply an upward shift in the time profile of extracted resource flows at each point in time.[5]

[4] The main role of the instantaneous death rate is to avoid the asymmetric dynamics and associated hysteresis effects that arise when entry entails a sunk cost. Such unnecessary complications would distract attention from the main point of the paper.

[5] If we model the resource stock $\Omega(t)$ as an exhaustible one, the harvesting rule would be $R(t) = \tilde{\rho}\Omega_0 e^{-\tilde{\rho}t}$,

2.4 Balanced trade

The asymmetric structure of trade implies that Home exhibits a structural deficit in manufacturing: being the sole supplier of resource-based intermediates, the value of its manufacturing imports necessarily exceeds the value of its manufacturing exports. We rule out trade in assets, so that trade in goods is balanced in each instant. Formally, then, the balanced-trade condition requiresχ

$$P_M M^F + \int_0^{N^H} P_i^H X_i^{Hf} di = \int_0^{N^F} P_i^F X_i^{Fh} di, \tag{14}$$

where the left-hand side is the value of total exports from Home to Foreign, i.e., the value of resource-based exports plus the value manufacturing exports, and the right-hand side is the value of manufacturing exports from Foreign to Home.

3 The World General Equilibrium

In this world economy there are $N^H + N^F + 1$ global goods markets: in each differentiated good market, the law of one price applies and the local monopolist sets the global price P_i^J given the global demand curve; in the market for the resource-based good M, the global price P_M is set competitively at the point where the collective supply of Home producers meets global demand. As the raw natural resource R is not tradeable, the Home's internal market for the resource clears when local demand meets local supply. The two labor markets clear separately in each country, determining the local wages. We rule out trade in assets so that there are two competitive financial markets: in each country the supply of investment funds from savers meets the demand from incumbent firms and entrepreneurs to determine the local interest rate. The no-arbitrage condition establishes that returns be equalized across alternative uses within each country, thereby determining the allocation of saving across incumbent firms and entrepreneurs.

3.1 Equilibrium Conditions

The equilibrium of the world economy is characterized by the following conditions (all derivations are in the Appendix). Each consumer allocates a fraction ξ of expenditure to domestic goods. Accordingly, the values of manufacturing production in each country are

$$Y^H = \xi E^H + (1 - \xi) E^F \quad \text{and} \quad Y^F = \xi E^F + (1 - \xi) E^H. \tag{15}$$

The time paths of expenditures are determined by the Keynes-Ramsey rules

$$\dot{E}^H / E^H = r^H - \rho \quad \text{and} \quad \dot{E}^F / E^F = r^F - \rho, \tag{16}$$

and the demand schedule for each manufacturing good is

$$X_i^J = \eth^J \cdot \left(P_i^J\right)^{-\epsilon}, \tag{17}$$

where $\tilde{\rho}$ is the reference discount rate, and an exogenous increase in Ω_0 would translate $R(t)$ upwards at any t. In the present model, we have a simpler but qualitatively identical effect since an upward jump in Ω corresponds to a proportional upward jump in R.

where $\eth^J \equiv Y^J / \int_0^{N^J} \left(P_i^J\right)^{1-\epsilon} di$. This term contains only aggregate variables and is therefore taken as given by each producer.

Each monopolist sets its price at a constant mark-up over marginal cost. Since the mark-up is invariant across varieties and firms choose identical intertemporal R&D strategies, the equilibrium is symmetric: in country J each monopolist employs the same amount of labor, $L_{Z_i}^J$, in raising its own productivity, as well as the same amounts of labor and resource-based input, $L_{X_i}^J$ and M_i^J, to produce the same quantity sold at the same price. Aggregating the conditional factor demands across firms yields

$$M^J = \sigma \frac{\epsilon - 1}{\epsilon} \frac{Y^J}{P_M}, \tag{18}$$

$$L_X^J = N^J \phi + (1 - \sigma) \frac{\epsilon - 1}{\epsilon} \frac{Y^J}{W^J}. \tag{19}$$

By symmetry, the typical firm's knowledge stock is $Z_i^J = Z^J$ and evolves according to

$$\frac{\dot{Z}^J}{Z^J} = \alpha \frac{L_Z^J}{N^J}, \tag{20}$$

where $L_Z^J = N^J L_{Z_i}^J$ is aggregate employment in vertical R&D. Symmetry also implies that the value of the firm is proportional to the value of its own production,

$$V_i^J = \beta \left(Y^J / N^J \right), \tag{21}$$

and that the equilibrium rate of horizontal innovation is

$$\frac{\dot{N}^J}{N^J} = \frac{W^J L_N^J}{\beta Y^J} - \delta. \tag{22}$$

The rates of vertical and horizontal innovation in (20) and (22) are interdependent through the no-arbitrage condition that the associated returns,

$$r_Z^J = \frac{\dot{W}^J}{W^J} + \alpha \left[\frac{Y^J}{N^J W^J} \theta \left(\frac{\epsilon - 1}{\epsilon} \right) - \frac{L_Z^J}{N^J} \right] - \delta, \tag{23}$$

$$r_N^J = \frac{\dot{Y}^J}{Y^J} - \frac{\dot{N}^J}{N^J} + \frac{1}{\beta} \left[\frac{1}{\epsilon} - \frac{N^J W^J}{Y^J} \left(\phi + \frac{L_Z^J}{N^J} \right) \right] - \delta, \tag{24}$$

must be equal. The financial market in each country clears when the resulting summary return to investment equals the return to assets demanded by savers in (16), i.e., when $r^J = r_Z^J = r_N^J$.

In country H the intermediate sector consists of an indefinite number of competitive firms that produce up to the point where the price equals the marginal cost. In equilibrium we thus have $P_M = C_M \left(W^H, p \right)$, where $C_M (.,.)$ is the unit cost function associated to (13). The conditional demands for the natural resource and labor read

$$pR = S_M^R \left(W^H, p \right) \cdot P_M M, \tag{25}$$

$$W^H L_M = \left(1 - S_M^R \left(W^H, p \right) \right) \cdot P_M M, \tag{26}$$

where

$$S_M^R \left(W^H, p \right) \equiv \frac{(1 - \varsigma)^\tau p^{1-\tau}}{(\varsigma)^\tau \left(W^H \right)^{1-\tau} + (1 - \varsigma)^\tau p^{1-\tau}} \tag{27}$$

is the elasticity of the unit-cost function with respect to the resource price. The resource market clears when $R = \Omega$. Notice that $S_M^R\left(W^H, p\right)$ is increasing in p if the natural resource and labor are complements ($\tau < 1$), decreasing in p if they are substitutes ($\tau > 1$), and independent of p in the special Cobb-Douglas case ($\tau = 1$). Combining (18) with (25) we obtain

$$p\Omega = S_M^R\left(W^H, p\right) \cdot \sigma \frac{\epsilon - 1}{\epsilon}\left(Y^H + Y^F\right). \tag{28}$$

This relation determines the price of the natural resource as a function of the value of world manufacturing production and Home's endowment of the resource.

Since the resource-processing sector only exists in country H, the market-clearing conditions for labor read $L^H = L_X^H + L_Z^H + L_N^H + L_M$ and $L^F = L_X^F + L_Z^F + L_N^F$, respectively. The set of equilibrium relations is then completed by the balanced-trade condition. In the Appendix we show that balanced trade implies that the values of expenditure and manufacturing production in the two countries are linearly interdependent. In particular:

$$\frac{E^F}{E^H} = \frac{1 - \sigma\frac{\epsilon-1}{\epsilon}}{1 + \frac{\xi}{1-\xi}\sigma\frac{\epsilon-1}{\epsilon}} \quad \text{and} \quad \frac{Y^F}{Y^H} = \frac{1}{1 + \frac{2\xi-1}{1-\xi}\sigma\frac{\epsilon-1}{\epsilon}}; \tag{29}$$

$$\frac{E^H}{Y^H} = \frac{1 + \frac{\xi}{1-\xi}\sigma\frac{\epsilon-1}{\epsilon}}{1 + \frac{2\xi-1}{1-\xi}\sigma\frac{\epsilon-1}{\epsilon}} \quad \text{and} \quad \frac{E^F}{Y^F} = 1 - \frac{\epsilon - 1}{\epsilon}\sigma. \tag{30}$$

It follows from (15) and (29) that balanced trade induces interest rate equalization since a constant expenditure ratio E^F/E^H implies $r^H = r^F$. This property suggests that our assumption of separated financial markets is indeed a simplification that has no substantial effect on the results.[6]

3.2 Instantaneous Equilibrium and Comparative Statics

We take as our numeraire the Home wage. This normalization implies that the equilibrium values of manufacturing production in both countries are constant at each point in time and is therefore equivalent to normalizing (nominal) expenditure.[7] This approach is standard in the trade literature, e.g., Grossman and Helpman (1991), and implies that the real growth rate of each economy is represented by the growth rate of the physical units appearing in the utility bundles (1)-(2). For expositional clarity, we study the determination of the equilibrium values of nominal expenditures in this section and study the associated growth rates in physical quantities in section 3.5.

Given $W^H \equiv 1$, the instantaneous equilibrium relations reduce to a system of four static equations determining simultaneously the resource price p, the values of manufacturing

[6] Results (29)-(30) hinge on the assumption that the manufacturing technology is Cobb-Douglas. If we posited a manufacturing technology with elasticity of factor substitution different from 1, e.g., a CES, the term σ in these expressions would be country-specific since it would be a function of the factor prices W^J and P_M^J. We could still characterize the world general equilibrium along the lines that we follow in the paper, but we would obtain essentially the same results at the cost of a vastly more complicated analysis.

[7] The normalization $W^H \equiv 1$ yields the static system (31)-(34) determining constant equilibrium values for manufacturing production. From (29)-(30), balanced trade implies constant ratios between expenditures and production in each country. As a consequence, the values of Home and Foreign expenditures, E^H and E^F, are constant over time, which implies $r^H = r^F = \rho$ from the Keynes-Ramsey rules (16).

production Y^H and Y^F, and the foreign wage W^F:

$$Y^H = L^H \frac{1+p\Omega}{1-\beta\rho+\frac{\epsilon-1}{\epsilon}\sigma\mu}; \quad (31)$$

$$Y^H = L^H \frac{1}{1-\beta\rho-f(p)}; \quad (32)$$

$$Y^F = \mu Y^H; \quad (33)$$

$$W^F = \frac{1}{L^F}\left(1-\beta\rho-\frac{\epsilon-1}{\epsilon}\sigma\right)Y^F, \quad (34)$$

where we have defined the constant $\mu \equiv 1/\left(1+\frac{2\xi-1}{1-\xi}\sigma\frac{\epsilon-1}{\epsilon}\right)$ and the function

$$f(p) \equiv \frac{\epsilon-1}{\epsilon}\sigma\left[(1+\mu)S_M^R(1,p)-\mu\right]. \quad (35)$$

We use subscripts '$*$' to denote the equilibrium values determined by (31)-(34). The system has the desirable property of being block-recursive, with equations (31)-(32) determining autonomously constant values Y_*^H and p_*, and the remaining two equations then determining constant values Y_*^F and W_*^F. We can thus proceed in steps.

Home Production and Resource Price. Equation (31) describes how manufacturing production per capita depends on the endowment per capita, via the latter's effect on resource income and thus on expenditure. This is a linear relation whereby an increase (decrease) in resource income, $p\Omega$, increases (decreases) the value of production via increased (reduced) expenditures. Equation (32) describes how the resource price, and hence resource income, depends on manufacturing production per capita via its effect on the demand for the natural resource. This relation is non-linear and its slope depends on the unit-cost elasticity $S_M^R(1,p)$ through the function $f(p)$ defined in (35). This implies that the properties of the equilibrium depend on whether the intermediate sector inputs, R and L_M, are complements, substitutes, or none of the two. The upper diagrams of Figure 1 illustrate the determination of the equilibrium values Y_*^H and p_* in the three cases. The lower diagrams describe the effects of exogenous variations in the resource endowment, Ω, on equilibrium values. When the resource endowment rises, the resource price falls in all cases but the effect on Y_*^H depends on the elasticity of substitution: manufacturing production per capita falls when $\tau < 1$, rises when $\tau > 1$, and remains the same when $\tau = 1$. The economic interpretation of these comparative-statics results is discussed in Proposition 1 below.

Foreign Production and Relative Wage. Due to international trade the value of Foreign's manufacturing production is directly related to Home's: (33) says that a change in Y_*^H yields a proportional change in Y_*^F; (34) then says that the change in Y_*^F yields a proportional change in W_*^F in the same direction. The key to these effects is that the change in Home's supply of the resource-based input requires a change in the same direction in the value of Foreign's manufacturing production since Foreign must increase the value of its manufacturing exports to pay for the higher value of its resource-based imports. The induced change in the demand for labor causes the Foreign wage to move in the same direction.

We summarize these results as follows.

Proposition 1 *Following an increase in Ω, the resource price p_* falls. Expenditure in Home, and expenditure and the wage in Foreign, move in the same direction as resource income in Home:*

$$sign\left(\frac{dY_*^H}{d\Omega}\right) = sign\left(\frac{dY_*^F}{d\Omega}\right) = sign\left(\frac{dW_*^F}{d\Omega}\right) = sign\ \left(\frac{d\left(\Omega p_*\right)}{d\Omega}\right),$$

where

$$\frac{d\left(\Omega p_*\right)}{d\Omega}\begin{cases} <0 \ \text{if}\ \tau<1 \\ =0 \ \text{if}\ \tau=1 \\ >0 \ \text{if}\ \tau>1 \end{cases}, \tag{36}$$

so that resource income rises (falls) if labor and the natural resource are substitutes (complements). If the resource-processing technology is Cobb-Douglas, the effect of the change of Ω on resource income is zero.

The intuition for these results is twofold. First, the elasticity of substitution in the intermediate sector determines the change in Home's resource income in response to an increase in the resource endowment: an increase in Ω always reduces the equilibrium price p_*, but the net effect on resource income, $d\left(\Omega p_*\right)/d\Omega$, depends on the elasticity of substitution between labor and the resource. If $\tau > 1$, the demand for the natural resource is elastic so that a positive shock to the resource supply yields a mild reduction of the equilibrium price. Accordingly, resource income rises and drives up equilibrium expenditure and production. In contrast, $\tau < 1$ implies inelastic demand, which yields a drastic reduction of the equilibrium resource price and a fall of resource income that drives down expenditure and production.

Second, the interdependence between the resource-rich and resource-poor economies due to balanced trade implies positive feedbacks: the value of production and expenditure in the two countries move in the same direction in response to the resource boom. The expansion/contraction of activity in Home induces an expansion/contraction in Foreign. A very important aspect of this interdependence is the effect on Foreign's wage. Since Foreign obtains resource-based intermediates by selling manufacturing goods to Home, an increase in Home's resource income generates a pressure on Foreign's wage through the increase in the demand for its tradeable final goods.

3.3 Innovation Rates

In both countries, aggregate productivity increases over time due to vertical and horizontal innovations. Substituting $r = \rho$ into (23) we obtain that the typical firm's knowledge stock evolves according to

$$\frac{\dot{Z}^J(t)}{Z^J(t)} = \begin{cases} \frac{Y_*^J}{W_*^J N^J(t)}\alpha\theta\frac{\epsilon-1}{\epsilon} - (\rho+\delta) & \text{if} \quad N^J(t) < \bar{N}^J \equiv \frac{\alpha\theta}{\rho+\delta}\frac{\epsilon-1}{\epsilon}\frac{Y_*^J}{W_*^J} \\ 0 & \text{if} \quad N^J(t) \geq \bar{N}^J \end{cases}. \tag{37}$$

This expression asserts that firms undertake vertical innovations according to the share $1/N^J$ of the wage-adjusted total market for manufacturing products, Y_*^J/W_*^J, that they capture: the larger the ratio $Y_*^J/\left(W_*^J N^J\right)$, the more they invest and the faster they grow. The wage adjustment reflects the fact that R&D is in units of labor and that it is an endogenous sunk cost that firms can spread over their own volume of production Y_*^J/N^J. If the mass of firms exceeds $\bar{N}^J$, the ratio $Y_*^J/\left(W_*^J N^J\right)$ falls below the critical threshold where they shut down R&D altogether.

Accordingly, the mass of firms evolves according to

$$\frac{\dot{N}^J(t)}{N^J(t)} = \begin{cases} \left[\frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\rho+\delta)\right]\left[1 - \frac{W_*^J}{Y_*^J}\frac{\frac{1}{\beta}\left(\phi - \frac{\rho+\delta}{\alpha}\right)}{\frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\rho+\delta)} N^J(t)\right] & \text{if} \quad N^J(t) < \bar{N}^J \\ \left[\frac{1}{\beta\epsilon} - (\rho+\delta)\right]\left[1 - \frac{W_*^J}{Y_*^J}\frac{\frac{\phi}{\beta}}{\frac{1}{\beta\epsilon} - (\rho+\delta)} N^J(t)\right] & \text{if} \quad N^J(t) \geq \bar{N}^J \end{cases}. \tag{38}$$

We concentrate on the first type of equilibria, where vertical innovation is positive, and assume that $N^J(t) < \bar{N}^J$ always holds (see the Appendix for details). A nice property of (38) is that it is a logistic equation that can be directly integrated to obtain the time path of the number of firms. Defining the coefficient $\nu \equiv \frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\rho+\delta)$, the explicit solution is

$$N^J(t) = \frac{N_{ss}^J}{1 + e^{-\nu t}\left(\frac{N_{ss}^J}{N_0^J} - 1\right)}, \tag{39}$$

where $N_0^J \equiv N^J(0)$ is given at $t = 0$. In the long run, we have:

$$\lim_{t\to\infty} N^J(t) = N_{ss}^J \equiv \frac{\alpha}{\epsilon}\frac{1-\theta(\epsilon-1) - \beta\epsilon(\rho+\delta)}{\alpha\phi - \rho - \delta}\frac{Y_*^J}{W_*^J}; \tag{40}$$

$$\lim_{t\to\infty} \frac{\dot{Z}^J(t)}{Z^J(t)} = z_{ss}^J \equiv \frac{\theta(\epsilon-1)(\alpha\phi - \rho - \delta)}{1-\theta(\epsilon-1) - \beta\epsilon(\rho+\delta)} - (\rho+\delta). \tag{41}$$

Results (40)-(41) imply that productivity growth in the long run is exclusively driven by vertical innovations. During the transition, the increase in firms' knowledge is accompanied by a process of net entry, or exit, in the manufacturing business. In the long run, the mass of firms converges to a constant equilibrium value and knowledge accumulation is the only source of productivity growth. Nevertheless, it would be misleading to conclude that horizontal innovation yields only a transitional effect on productivity growth. In fact, it plays a crucial role in determining the effect of endowments. The important property of the limit z_{ss}^J is that it is independent of the endowments Ω, L^H and L^J because changes in endowments trigger changes in the wage-adjusted value of manufacturing production, Y_*^J/W_*^J, which are fully absorbed by the mass of firms, N_{ss}^J, as (40) shows. This result highlights the general absence of scale effects in the class of models that allow for both vertical and horizontal innovation. Moreover, exploiting the closed-form solutions for the paths of the key state variables N^H and N^F, we can characterize in detail how each economy adjusts to the resource boom.

Innovation rates in Home. Assume that Ω increases suddenly at time t_0 and, for simplicity, that the economy is in steady state at t_0. Consider first the case $\tau > 1$. From Proposition 1, the resource boom causes an immediate and permanent increase in Y_*^H. From (40), this increase in market size yields an increase in the steady-state mass of firms N_{ss}^H. From (38), the rate of net entry $\dot{N}^H/N^H$ jumps up and then declines, taking the economy gradually and smoothly from $N^H(t_0)$ to the new steady state. From (37), the upward jump in expenditure Y_*^H yields an initial jump up of the rate of vertical innovation $\dot{Z}^H/Z^H$. As the mass of firms grows during the transition the effect of the larger market size is absorbed and in the long run the value of the production of each firm, $Y_*^H/N^H(t)$, returns to the same steady-state level as before the shock. Consequently, the rate of vertical innovation also returns to the steady-state level in (41) which is independent of the endowments L^H and Ω. The direction of these effects is reversed when $\tau < 1$. In this case, the resource boom induces a drop in Y_*^H and thereby in N_{ss}^H. During the transition there is net exit from the manufacturing business, $\dot{N}^H < 0$, which drives the rate of vertical innovation back to the pre-shock level z_{ss}^H.

Innovation rates in Foreign. The Foreign economy adjusts to the resource boom in Home by simply changing its volume of production and its wage without exhibiting any change in the dynamics of innovation rates. The crucial equation for this result is (34), showing that the ratio between equilibrium expenditure and wage is constant and independent of Home's resource endowment. If $\tau > 1$ ($\tau < 1$) the resource boom increases (decreases) Foreign expenditure Y_*^F via trade — see (33) and Proposition 1 — but the Foreign wage W_*^F increases (decreases) proportionally. From (38), a constant ratio Y_*^F/W_*^F implies no effects on the rate of horizontal innovation $\dot{N}^F/N^F$, and thereby unchanged dynamics in the rate of vertical innovation $\dot{Z}^F/Z^F$.

We summarize these results as follows.

Proposition 2 *If $\tau > 1$ ($\tau < 1$), an increase in Ω generates a higher (lower) steady-state level of the mass of firms in the Home economy, which increases (decreases) both the rates of horizontal and vertical innovation during the transition. In the long run, both innovation rates in Home converge to the same steady-state levels as before the shock, i.e., $\dot{N}^H/N^H = 0$ and $\dot{Z}^H/Z^H = z_{ss}^H$. The Foreign economy fully absorbs the shock by increasing (decreasing) the wage, with no effects on transitional or long-run innovation rates.*

In the next subsection, we exploit the closed-form solutions derived above to determine the exact behavior of sectoral employment levels during the transition.

3.4 The Reallocation of Labor in Home

The effects of the resource boom on innovation rates hinge on a reallocation mechanism that changes the employment shares of the various economic activities. This reallocation follows directly from the market-clearing conditions (see Appendix):

$$L_N^H(t) = \delta\beta Y_*^H + \frac{1-\theta(\epsilon-1)-\epsilon\beta(\rho+\delta)}{\epsilon}Y_*^H - \frac{\alpha\phi-\rho-\delta}{\alpha}N^H(t); \tag{42}$$

$$L_Z^H(t) = Y_*^H\theta\frac{\epsilon-1}{\epsilon} - \frac{\rho+\delta}{\alpha}N^H(t); \tag{43}$$

$$L_X^H(t) = \phi N^H(t) + (1-\sigma)\frac{\epsilon-1}{\epsilon}Y_*^H; \tag{44}$$

$$L_M(t) = L^H - \left(1-\sigma\frac{\epsilon-1}{\epsilon}\right)Y_*^H. \tag{45}$$

Assume that Ω increases at time t_0 and consider first the case $\tau > 1$. Recall that the mass of firms at the time of the shock, $N^H(t_0)$, is pre-determined and does not jump. From Proposition 2, the resource boom yields an immediate increase in Y_*^H and the effects on Home's allocation of labor are as follows.

>From (42), employment in start-up activities, $L_N^H(t)$, jumps up at time t_0. Higher expenditure moves labor into entry operations that drive the gradual increase in the mass of firms $N^H(t)$ towards N_{ss}^H. Importantly, after the inital upward jump, employment in entry gradually declines but converges to a higher steady-state with respect to the pre-shock steady-state level (see Appendix). Similar dynamics characterize the transition of $L_Z^H(t)$. From (43), labor employed in vertical innovation is positively related to expenditures but negatively related to the mass of firms: $L_Z^H(t)$ jumps up with Y_*^H at t_0 and then gradually declines as $N^H(t)$ grows. The initial jump is not fully absorbed by entry, however, and the new steady-state level of $L_Z^H(t)$ is higher than its pre-shock level.

Labor employed in manufacturing production reacts differently. From (44), $L_X^H(t)$ jumps up with Y_*^H at t_0 and, after the initial jump, keeps rising as the entry process attracts even more labor into production. Hence, $L_X^H(t)$ converges to a level that exceeds the pre-shock steady state for two reasons: (a) each firm wishes to produce more because the market is larger and (b) the larger market supports more firms.

Finally, equation (45) shows that total labor employed in the resource-processing sector is negatively related to expenditure and independent of the mass of firms: $L_M(t)$ adjusts instantaneously to the sudden increase in Y_*^H due to the endowment shock with no further dynamics.

We summarize all these results in Figure 2, left graphs. The effects of a resource boom in the opposite case $\tau < 1$ obviously yield specular transition paths (Figure 2, right graphs).

These results suggest two remarks. First, the two forces driving this transition — the immediate increase in market size and the process of net entry it induces — offset each other in the long run so that employment per firm in production, $L_X^H(t)/N^H(t)$, and vertical innovation, $L_Z^H(t)/N^H(t)$, is independent of the endowments L^H and Ω. This exact offset — which is the key to the elimination of the scale effect — is the reason why the growth acceleration generated by the resource boom is only temporary. Second, the crowding-in (crowding-out) effects generated by resource booms in the manufacturing sector are self-reinforcing due to the two, interdependent dimensions of innovation: after the initial jump, the process of net entry (net exit) yields further increases (decreases) of total labor employed in final production along the transition to the new long-run equilibrium.

3.5 Total Factor Productivity, Growth and Welfare

The closed-form solutions derived above allow us to characterize analytically the relative growth performance of trading economies and the welfare effects of resource booms. Substituting the equilibrium relations obtained in the utility functions (1) and (2), instantaneous utility levels equal (see Appendix)

$$\log u^H(t) = \log \Upsilon_*^H + \log T^H(t)^{\xi} T^F(t)^{1-\xi}; \tag{46}$$

$$\log u^F(t) = \log \Upsilon_*^F + \log T^F(t)^{\xi} T^H(t)^{1-\xi}. \tag{47}$$

Expressions (46)-(47) decompose utility levels in two terms. The first is represented by the intercept Υ_*^J, which is constant over time and depends on the equilibrium values of expenditures, wages and resource price:

$$\Upsilon_*^H \equiv \left[\frac{\chi^H \frac{\epsilon-1}{\epsilon} Y_*^H}{L^H C_X^H (1, C_M (1, p_*))}\right]^{\xi} \left[\frac{\chi^F \frac{\epsilon-1}{\epsilon} Y_*^F}{L^F C_X^F (W_*^F, C_M (1, p_*))}\right]^{1-\xi}, \tag{48}$$

$$\Upsilon_*^F \equiv \left[\frac{\left(1-\chi^H\right) \frac{\epsilon-1}{\epsilon} Y_*^H}{L^H C_X^H (1, C_M (1, p_*))}\right]^{\xi} \left[\frac{\left(1-\chi^F\right) \frac{\epsilon-1}{\epsilon} Y_*^F}{L^F C_X^F (W_*^F, C_M (1, p_*))}\right]^{1-\xi}, \tag{49}$$

where χ^H and χ^F are exogenous constants defined in the Appendix. The second term is time-varying and is the weighted average of the total factor productivity indices of both economies defined as:

$$T^H(t) \equiv \left(Z^H(t)\right)^{\theta} \left(N^H(t)\right)^{\frac{1}{\epsilon-1}} \quad \text{and} \quad T^F(t) \equiv \left(Z^F(t)\right)^{\theta} \left(N^F(t)\right)^{\frac{1}{\epsilon-1}}.$$

The weights associated to the aggregate TFP indices in (46)-(47) are ξ and $1-\xi$, that is, the parameters representing preferences for domestic and imported final goods. This result clarifies that the growth rate of *each* economy depends on the total factor productivity growth of *both* economies due to trade. If productivity growth in one economy, say Home, is affected by a country-specific shock, real income growth and welfare dynamics in Foreign respond even though Foreign TFP growth is not affected by the shock.

Equations (46)-(47) allow us to split the overall welfare effect of a resource boom into (i) level effects at t_0 and (ii) transitional growth effects after t_0. The instantaneous level effects are captured by the intercept terms Υ_*^H and Υ_*^F: from Proposition 1, a sudden increase in Ω generates immediate jumps in equilibrium expenditures, the relative wage and the resource price $(Y_*^H, Y_*^F, W_*^F, p^*)$. Since the firm-specific knowledge stocks (Z^J) and the mass of firms (N^J) in both countries are fixed at time t_0, the instantaneous effects of the resource boom on $u^J(t_0)$ are only due to the jumps of Υ_*^H and Υ_*^F. The transitional growth effects arising after t_0, instead, are due to the dynamics of the TFP indices. As shown in Proposition 1, a sudden increase in Ω modifies innovation rates only in Home. The resulting transitional dynamics in Home's total factor productivity $T^H(t)$ affect, in turn, the growth rates of utility in *both* economies.

The fact that the mass of firms in each country follows a logistic process allows us to characterize the behavior of TFP in closed form.

Proposition 3 *In each country the behavior of TFP levels over time is given by*

$$\log T^J(t) = \log T_0^J + g_{ss}^J \cdot t + \left(\frac{\gamma}{\nu} + \frac{1}{\epsilon-1}\right) \Delta^J \cdot \left(1 - e^{-\nu \cdot t}\right), \tag{50}$$

where

$$\gamma \equiv \theta\alpha\theta \left(\frac{\epsilon-1}{\epsilon}\right) \frac{Y_*^J}{W_*^J N_{ss}^J} = \frac{\alpha\theta^2 (\epsilon-1) \left(\phi - \frac{\rho+\delta}{\alpha}\right)}{1 - \theta(\epsilon-1) - \beta\epsilon(\rho+\delta)},$$

and

$$\Delta^J \equiv \frac{N_{ss}^J}{N_0^J} - 1$$

is a country-specific value that remains constant throughout the transition.

Proof. See the Appendix. ■

The time paths of $T^H(t)$ and $T^F(t)$ capture the combined effects of a resource boom on market size and knowledge accumulation in each economy. To see the effects on welfare in Home, we substitute result (50) in (46) and calculate present-value utility U_0^H from (3), obtaining

$$\begin{aligned} \rho U_0^H &= \log \left(T_0^H\right)^{\xi} \left(T_0^F\right)^{1-\xi} + \\ &+ \underbrace{\log \Upsilon_*^H}_{\text{Static Term}} + \underbrace{\frac{1}{\rho}\left[\xi g_{ss}^H + (1-\xi)\, g_{ss}^F\right]}_{\text{Long-run Growth Term}} + \Phi \underbrace{\left[\xi \Delta^H + (1-\xi)\,\Delta^F\right]}_{\text{Transitional Growth Term}}, \end{aligned} \tag{51}$$

where

$$\Phi \equiv \frac{\gamma + \frac{\nu}{\epsilon - 1}}{\rho + \gamma}.$$

Expression (51) decomposes welfare in Home in three parts. The static term captures the baseline value of instantaneous utility in (46), which is a weighted average of the expenditure-to-production-cost ratios observed in the two countries. The second term is a weighted average of the long-run growth rates in the two countries. The third term is a weighted average of the changes in market size Δ^J determining the transitional growth rates in the two countries. Since an exogenous increase in Ω does not modify long-run growth rates, resource booms affect welfare through variations in the static term (static effects) and in the transitional growth term (transitional effects). Specifically:

i. From (48), the static effect $\partial \Upsilon_*^H / \partial \Omega$ has two components. The first component says that when Ω rises, p_* in the denominator falls while Y_*^H rises (falls) if $\tau > 1$ ($\tau < 1$). The second component is slightly more complicated because there is an adverse wage effect that tends to offset the effect of the drop in the resource price: W_*^F moves in the same direction as Y_*^H. Whether foreign goods become more or less expensive depends on the relative strength of the two effects. Consequently, the sign of the static effect is generally ambiguous.

ii. Since Δ^F is independent of Ω, the transitional effect of a resource boom shows up only through Δ^H, which moves in the same direction as Y_*^H and therefore rises (falls) if $\tau > 1$ ($\tau < 1$).

Given the quasi-symmetry of the model, the expression for welfare in Foreign is qualitatively identical. We thus don't show it to save space. The difference is that the change in welfare due to the resources boom is fully 'imported'.

Expression (51) allows us to identify a clear trade-off that does not arise in the closed-economy version of this model (Peretto 2008). To see it most clearly, consider the case of substitution, which yields an elastic resource demand. In closed economy, $\tau > 1$ yields a welfare increase due to an upward jump in utility at time 0, followed by a temporary growth acceleration that eventually dies out. In this paper, we have a similar 'Home effect' which is however accompanied by a 'Foreign effect' induced by international trade. Using (48) and rearranging terms in (51), we have

$$\rho U_0^H = \log\left(T_0^H\right)^{\xi}\left(T_0^F\right)^{1-\xi} + \underbrace{\xi \cdot \left\{\log\left[\frac{\chi^{H}\frac{\epsilon-1}{\epsilon}(Y_*^H/L^H)}{C_X^H\left(1, C_M\left(1,p_*\right)\right)}\right] + \frac{1}{\rho}g_{ss}^H + \Phi\Delta^H\right\} +}_{\text{Home Term}}$$

$$\underbrace{+(1-\xi)\cdot\left\{\log\left[\frac{\chi^{F}\frac{\epsilon-1}{\epsilon}(Y_*^F/L^F)}{C_X^F\left(W_*^F, C_M\left(1,p_*\right)\right)}\right] + \frac{1}{\rho}g_{ss}^F + \Phi\Delta^F\right\}}_{\text{Foreign Term}}.$$

As Ω rises, $\tau > 1$ implies that the Home term increases due to a positive static effect, $d(Y_*^H/C_X^H)/d\Omega > 0$, and to a positive transitional effect, $d\Delta^H/d\Omega > 0$. The Foreign term, however, may react in either direction since the sign of

$$\frac{d}{d\Omega}\log\left[\frac{\chi^{F}\frac{\epsilon-1}{\epsilon}(Y_*^F/L^F)}{C_X^F\left(W_*^F, C_M\left(1,p_*\right)\right)}\right] \tag{52}$$

is generally ambiguous. Following a resource boom with $\tau > 1$, if the increase in the Foreign wage is weak relative to the reduction of the resource price, unit production costs in Foreign fall and (52) is positive. In this case, the Foreign effect reinforces the Home effect and the overall effect on Home welfare is positive. If the wage increase dominates the fall in p_*, instead, (52) is negative and the Foreign Effect contrasts the Home effects.

The role of international trade is evident also in the case of complementarity. When $\tau < 1$, the resource boom triggers such a drastic fall of p_* that expenditure Y_*^H falls as well, and generates a growth slowdown – the so-called Curse of Natural Resources. If the economy is closed, the curse manifests itself as a *temporary* slowdown as the economy returns to the same steady-state growth rate as before the shock. In our open economy model, the curse manifests itself also in the Foreign economy. The reason is that the fall of Home expenditure Y_*^H triggers a fall of Foreign expenditure Y_*^F which in turn triggers a fall of its wage.

4 Discussion

As noted in the Introduction, the theoretical literature on international trade and innovation-driven economic growth usually abstracts from asymmetric trade structures induced by uneven endowments in specific primary factors. It focuses on convergence issues and catching-up processes whereby lagging economies may reduce structural growth differentials with technology leaders through knowledge spillovers (Grossman and Helpman 1991), flows of ideas (Rivera-Batiz and Romer 1991), imitation (Barro and Sala-i-Martin 1997) or trade in intermediate inputs (Feenstra 1996). Our analysis adopts a similar two-country framework but focuses on the relative performance between resource-rich and resource-poor economies and the correlation between resource abundance, income levels and growth.

The analysis of resource-rich economies is typically associated with the parallel literature on Dutch-Disease phenomena and/or the Curse of Natural Resources. The empirical case for the resource curse hypothesis builds on the results of Sachs and Warner (1995) who observed that many resource-rich countries display slow growth. Several theoretical studies explain the curse by following the logic of Dutch Disease models (Corden, 1984): they characterize resource-rich countries as small open economies trading resource-based commodities at fixed international prices, and interpret the resource curse as a productivity slowdown generated by *sectoral booms* — sudden increases in resource incomes due to exogenous shocks that raise the size or the productivity of the resource-intensive sector in the economy. The conventional view is that these booms harm economic growth because the reallocation of labor and capital toward the resource intensive sector results into the crowding-out of strategic, knowledge-creating sectors (Van Wijnbergen, 1984; Krugman, 1987; Gylfason et al., 1999; Torvik, 2001).

Our analysis differs from the resource-curse literature in several ways. At the conceptual level, we do not attempt to explain the resource curse: in our model greater resource abundance may yield slower or faster growth, but the central aim of the analysis is to characterize in detail the transmission channels between shocks to resource endowments, income levels and economic growth in the presence of asymmetric trade and endogenous innovations. At the formal level, three main aspects differentiate our analysis from Dutch-Disease models.

First, Dutch Disease models do not distinguish between physical resource endowments and resource wealth: resource booms are characterized as sudden increases in the relative profitability of the primary sector generated by technological shocks, increases in the world price of the resource, or exogenous improvements in terms of trade. Unexpected discoveries of new resource endowments are treated in the same way as an "exchange rate gift" (e.g., Torvik, 2001) but this is admittedly an approximation: studying the correlation between natural endowments and economic growth requires distinguishing between physical resource endowments and flows of resource income. Second, in our model, primary sectors of resource-rich countries are vertically related to both domestic and foreign final sectors, and the elasticity of demand for intermediates is the crucial transmission channel between endowment shocks and aggregate productivity. This aspect is generally neglected in Dutch-Disease models since they ignore vertically related markets. Third, Dutch-Disease models view resource-rich countries as small open economies. If the world price of the resource is exogenously fixed, a sudden increase in the resource endowment immediately translates into a rise in the value of the resource and thereby into an upward shift of the time-profile of resource rents. This mechanism, however, neglects the role of price effects that, as our analysis emphasizes, determine the extent to which resource-rich economies are able to exploit the natural endowment to obtain additional income. Interestingly, in his seminal paper Corden (1984) regards the lack of price effects as a simplifying assumption: "extra exports of [the resource-intensive good] owing to technical progress in the [resource-intensive] sector or any other reason may lower the world price of [the resource-intensive good]. This is an obvious effect, and we can suppose that it has already been incorporated in the calculation of the size of the boom." (Corden, 1984: p.367). The subsequent literature, however, does not extend the basic framework to include price effects via endogenous terms of trade.

These differences in assumptions are reflected in our results. In Dutch-Disease models based on small open economies (Van Wijnbergen, 1984; Krugman, 1987; Gylfason et al., 1999; Torvik, 2001), greater resource abundance is associated with (i) higher resource income, (ii) higher employment in the resource-intensive sector, (iii) less knowledge creation and slower growth. In our model, instead, resource income may increase or decrease after a resource boom and the resulting dynamics differ. Specifically, we show that under substitutability the resource boom generates (i) higher resource income, (ii) lower employment in the resource-intensive sector, (iii) higher knowledge creation and faster growth. Under complementarity, instead, a resource boom generates (i) lower resource income, (ii) higher employment in the resource-intensive sector, (iii) less knowledge creation and slower growth. Hence, our model predicts negative growth effects of resource abundance only if resource incomes are reduced by the resource boom.

5 Conclusion

In this paper we studied a two-country model of R&D-based growth featuring both inter-industry and intra-industry trade due to uneven natural resource endowments. The resource-rich economy, that we call Home, exports resource-based intermediates and final manufacturing goods and imports differentiated final goods from the resource-poor economy, that we call Foreign. Both economies develop innovations that generate endogenous productivity growth. In this framework, we analyze the effects of *resource booms* — sudden increases in Home's natural resource endowment due to unexpected discoveries — on expenditure levels, innovation rates, productivity growth, the allocation of labor across sectors, and welfare levels in both economies.

A sudden rise in Home's resource endowment induces a change in the demand for manufacturing goods produced by Foreign, which triggers a change in the relative wage but leaves Foreign's TFP growth unaffected. In Home, instead, labor is reallocated across primary, manufacturing and R&D activities, with permanent effects on the equilibrium number of firms and transitional effects on TFP growth. The balanced trade condition then implies that the variation in TFP growth experienced by Home shows up in Foreign's import price index so that, due to trade, the dynamic effects of the resource boom are transmitted to the resource-poor country.

The sign of growth effects crucially depends on whether labor and the raw resource are complements or substitutes in the production of resource-based intermediates. If there is substitutability, a resource boom raises Home's resource income and its overall expenditure on manufacturing goods. In Foreign, the wage increases due to Home's higher demand for manufacturing goods. In Home, labor is reallocated towards manufacturing and there is a permanent positive effect on the equilibrium number of firms: during the transition, the process of entry in the manufacturing business raises Home's TFP growth via horizontal innovations. This positive growth effect in Home is then transmitted to Foreign through trade, since households in each country consume both domestic and imported goods. If there is complementarity, instead, the same mechanism works in the opposite direction: a positive shock to Home's resource endowment drives down resource income and thereby Home's demand for Foreign's manufacturing goods. Consequently, Foreign's wage falls, Home's number of firms converges towards a lower steady-state level and transitional growth effects are negative for both countries. The intuition behind these results is that the elasticity of substitution in the intermediate sector determines the reaction of Home's resource income to an increase in the resource endowment. Substitutability implies elastic demand for the raw resource, so that a positive shock to the resource supply requires a mild reduction in the resource price and the net effect on resource income is positive. Complementarity, instead, implies inelastic demand, which means that the resource boom causes fall of the resource price so drastic that it more than offsets the larger quantity sold with the result that resource income falls.

The transitional dynamics of sectoral labor employment in the Home economy are driven by the coexistence of vertical and horizontal innovations. When the resource boom yields higher (lower) resource income, the first reallocation effect is a sudden increase (reduction) in labor employed in final production due to the upward (downward) jump in expenditures. Subsequently, the process of net entry (net exit) of firms in the manufacturing business implies further increases (decreases) in total labor employed in producing final goods in Home. Hence, the crowding-in (crowding-out) effects generated by resource booms are self-reinforcing due to the two interdependent dimensions of innovation.

Our analysis suggests three questions that deserve empirical scrutiny. First, the response of primary employment to resource-endowment booms may substantially differ from the response to resource-income booms: to our knowledge, there is no empirical analysis dedicated to this issue. Second, in line with the results of Lederman and Maloney (2007), asymmetric trade matters for growth and, given the existing interdependencies, the empirical analysis could be further extended to investigate how the economic performance of resource-poor countries responds to resource booms. Third, the central role of the elasticity of substitution between resources and labor in our model suggests analyzing in detail whether regular technological biases exist in the production process of resource-based industries.

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A Appendix

Derivation of (15). Each household solves the static problem

$$\max_{\left\{X_i^{Hj}, X_i^{Fj}\right\}} \log u^J \text{ s.t. } E^J/L^J = \int_0^{N^H} \left(P_i^H X_i^{Hj}/L^J\right) di + \int_0^{N^F} \left(P_i^F X_i^{Fj}/L^J\right) di,$$

where $J = H, F$, $j = h, f$, and $\log u^J$ is defined by (1)-(2). Denoting by $\varkappa^H$ the Lagrange multiplier, the first-order conditions in H are

$$X_i^{Hh} = \frac{L^H \left(P_i^H\right)^{-\epsilon} \xi^\epsilon}{\left[\varkappa^H \int_0^{N^H} \left(X_i^{Hh}/L^H\right)^{\frac{\epsilon-1}{\epsilon}} di\right]^\epsilon} \text{ and } X_i^{Fh} = \frac{L^H \left(P_i^F\right)^{-\epsilon} (1-\xi)^\epsilon}{\left[\varkappa^H \int_0^{N^F} \left(X_i^{Fh}/L^H\right)^{\frac{\epsilon-1}{\epsilon}} di\right]^\epsilon}. \tag{A.1}$$

Multiplying both sides of the first (second) equation by P_i^H (P_i^F), integrating both sides across varieties, and eliminating $\varkappa^H$ by means of the initial expressions in (A.1), we obtain

$$X_i^{Hh} = \frac{\int_0^{N^H} P_i^H X_i^{Hh} di}{\int_0^{N^H} \left(P_i^H\right)^{1-\epsilon} di} \left(P_i^H\right)^{-\epsilon} \text{ and } X_i^{Fh} = \frac{\int_0^{N^F} P_i^F X_i^{Fh} di}{\int_0^{N^F} \left(P_i^F\right)^{1-\epsilon} di} \left(P_i^F\right)^{-\epsilon}. \tag{A.2}$$

Taking the ratio between these two expressions and substituting X_i^{Hh} and X_i^{Fh} by means of (A.1), we have

$$\frac{\int_0^{N^H} P_i^H X_i^{Hh} di}{\int_0^{N^F} P_i^F X_i^{Fh} di} = \frac{\xi}{1-\xi}. \tag{A.3}$$

Following the same steps for F, we have

$$X_i^{Hf} = \frac{\int_0^{N^H} P_i^H X_i^{Hf} di}{\int_0^{N^H} \left(P_i^H\right)^{1-\epsilon} di} \left(P_i^H\right)^{-\epsilon} \text{ and } X_i^{Ff} = \frac{\int_0^{N^F} P_i^F X_i^{Ff} di}{\int_0^{N^F} \left(P_i^F\right)^{1-\epsilon} di} \left(P_i^F\right)^{-\epsilon}, \tag{A.4}$$

and

$$\frac{\int_0^{N^H} P_i^H X_i^{Hf} di}{\int_0^{N^F} P_i^F X_i^{Ff} di} = \frac{1-\xi}{\xi}. \tag{A.5}$$

Market clearing yields the values of production in the two countries:

$$Y^H = \int_0^{N^H} P_i^H X_i^{Hh} di + \int_0^{N^H} P_i^H X_i^{Hf} di; \tag{A.6}$$

$$Y^F = \int_0^{N^F} P_i^F X_i^{Fh} di + \int_0^{N^F} P_i^F X_i^{Ff} di. \tag{A.7}$$

>From (A.3) and (4), we have $\int_0^{N^H} P_i^H X_i^{Hh} di = \xi E^H$ and $\int_0^{N^F} P_i^F X_i^{Fh} di = (1-\xi) E^H$; from (A.5) and (5), we have $\int_0^{N^H} P_i^H X_i^{Hf} di = (1-\xi) E^F$ and $\int_0^{N^F} P_i^F X_i^{Ff} di = \xi E^F$. Combining these with the constraints (A.6)-(A.7), we obtain (15).

Derivation of (16). Substituting (A.3) in (1) and (A.5) in (2), we obtain the indirect utility

$$\tilde{u}^J \left(E^J/L^J\right) = \log \xi^\xi (1-\xi)^{1-\xi} + \log \left(E^J/L^J\right). \tag{A.8}$$

In each country the household chooses the time path of expenditure $\left(E^J/L^J\right)$ that maximizes

$$\int_0^\infty e^{-\rho t} \tilde{u}^J \left(E^J(t)/L^J\right) dt$$

subject to the appropriate wealth constraint — (6) or (7) — re-written in terms of assets per capita. The logarithmic form (A.8) implies the standard Keynes-Ramsey rules (16).

Derivation of (17). From (A.2) and (A.4), we have:

$$\frac{X_i^{Hh}}{X_i^{Hf}} = \frac{\int_0^{N^H} P_i^H X_i^{Hh} di}{\int_0^{N^H} P_i^H X_i^{Hf} di} = \frac{\xi}{1-\xi} \cdot \frac{E^H}{E^F}; \tag{A.9}$$

$$\frac{X_i^{Fh}}{X_i^{Ff}} = \frac{\int_0^{N^F} P_i^F X_i^{Fh} di}{\int_0^{N^F} P_i^F X_i^{Ff} di} = \frac{1-\xi}{\xi} \cdot \frac{E^H}{E^F}, \tag{A.10}$$

where the last terms follow from the derivation of (15) above. Hence:

$$X_i^{Hh} + X_i^{Hf} = X_i^{Hh} \left[1 + \frac{1-\xi}{\xi} \cdot \frac{E^F}{E^H}\right]; \tag{A.11}$$

$$X_i^{Ff} + X_i^{Fh} = X_i^{Ff} \left[1 + \frac{1-\xi}{\xi} \cdot \frac{E^H}{E^F}\right]. \tag{A.12}$$

Using (A.2) and (A.4) to eliminate, respectively, X_i^{Hh} and X_i^{Ff} from the right-hand sides of (A.11) and (A.12), we obtain:

$$X_i^{Hh} + X_i^{Hf} = \frac{\xi E^H + (1-\xi) E^F}{\int_0^{N^H} \left(P_i^H\right)^{1-\epsilon} di} \left(P_i^H\right)^{-\epsilon} = \left(P_i^H\right)^{-\epsilon} \left[\frac{Y^H}{\int_0^{N^H} \left(P_i^H\right)^{1-\epsilon} di}\right]; \tag{A.13}$$

$$X_i^{Ff} + X_i^{Fh} = \frac{\xi E^F + (1-\xi) E^H}{\int_0^{N^F} \left(P_i^F\right)^{1-\epsilon} di} \left(P_i^F\right)^{-\epsilon} = \left(P_i^F\right)^{-\epsilon} \left[\frac{Y^F}{\int_0^{N^F} \left(P_i^F\right)^{1-\epsilon} di}\right], \tag{A.14}$$

where the left-hand sides represent the total demand for the i-th variety produced in country H and F, respectively. Substituting $X_i^H = X_i^{Hh} + X_i^{Hf}$ and $X_i^F = X_i^{Ff} + X_i^{Fh}$ in (A.13) and (A.14), respectively, we obtain the demand schedule (17).

The monopolist problem. The producer of the i-th variety in country J solves the following problem. Given technology (8), the cost-minimizing conditions over rival inputs, $L_{X_i}^J$ and M_i^J, yield $\frac{W^J}{P_M} = \frac{\sigma}{1-\sigma}\frac{M_i^J}{L_{X_i}^J-\phi}$, which in turn yields total cost

$$W^J L_{X_i}^J + P_M M_i^J = W^J\phi + C_X^J\left(W^J, P_M\right)\cdot\left(Z_i^J\right)^{-\theta} X_i^J, \tag{A.15}$$

where

$$C_X^J\left(W^J, P_M\right) \equiv \left(P_M\right)^{\sigma}\left(W^J\right)^{1-\sigma}\left[\left(\frac{\sigma}{1-\sigma}\right)^{\sigma} + \left(\frac{1-\sigma}{\sigma}\right)^{1-\sigma}\right] \tag{A.16}$$

is the standard unit-cost function homogeneous of degree one. From (A.15) instantaneous profits $\Pi_{X_i}^J$ read $\left[P_i^J - C_X^J\left(W^J, P_M\right)\cdot\left(Z_i^J\right)^{-\theta}\right] X_i^J - W^J\phi - W^J L_{Z_i}^J$, where $W^J L_{Z_i}^J$ is R&D expenditure. Since the monopolist knows the demand schedule (17), instantaneous profits can be written as

$$\Pi_{X_i}^J = \left[P_i^J - C_X^J\left(W^J, P_M\right)\cdot\left(Z_i^J\right)^{-\theta}\right]\eth^j\left(P_i^J\right)^{-\epsilon} - W^J\phi - W^J L_{Z_i}^J. \tag{A.17}$$

where $\eth^J$ is taken as given by the single monopolist since it contains only aggregate variables. The problem of the firm is to maximize $V_i^J(t)$ defined in (11), with instantaneous profits given by (A.17), subject to the knowledge-accumulation law $\dot{Z}_i^J = \alpha K^J \cdot L_{Z_i}^J$ with aggregate knowledge K^J taken as given. The current-value Hamiltonian associated with this problem is

$$\mathcal{L} = \left[P_i^J - C_X^J\left(W^J, P_M\right)\cdot\left(Z_i^J\right)^{-\theta}\right]\left(P_i^J\right)^{-\epsilon}\cdot\eth^j - W^J\phi - W^J L_{Z_i}^J + \zeta_i^J\alpha K^J L_{Z_i}^J,$$

where P_i^J and $L_{Z_i}^J$ are control variables, and Z_i^J is the state variable associated with the dynamic multiplier ζ_i^J. The necessary conditions for optimality are:

$$\partial\mathcal{L}/\partial P_i^J = 0 \rightarrow X_i^J = \epsilon\left[P_i^J - C_X^J\left(W^J, P_M\right)\cdot\left(Z_i^J\right)^{-\theta}\right]\left(P_i^J\right)^{-\epsilon-1}\eth^j; \tag{A.18}$$

$$\partial\mathcal{L}/\partial L_{Z_i}^J = 0 \rightarrow \zeta_i^J\alpha K^J - W^J \leq 0 \ (= 0 \text{ if } L_{Z_i}^J > 0); \tag{A.19}$$

$$\partial\mathcal{L}/\partial Z_i^J = \left(r^J + \delta\right)\zeta_i^J - \dot{\zeta}_i^J; \tag{A.20}$$

$$0 = \lim_{s\rightarrow\infty} e^{-\int_t^s\left[r^J(v)+\delta\right]dv}\zeta_i^J(s)\, Z_i^J(s). \tag{A.21}$$

Condition (A.18) determines the price-setting rule of each monopolist,

$$P_i^J = \frac{\epsilon}{\epsilon-1}C_X^J\left(W^J, P_M\right)\cdot\left(Z_i^J\right)^{-\theta}, \tag{A.22}$$

which implies a positive mark-up of over the marginal cost. Condition (A.19) shows that, in an interior solution $L_{Z_i}^J > 0$, the marginal cost W^J must equal the marginal benefit of knowledge accumulation. In an interior solution $\zeta_i^J\alpha K^J = W^J$, the co-state equation (A.20) implies

$$\frac{\dot{\zeta}_i^J}{\zeta_i^J} = r^J + \delta - \frac{\theta C_X^J\left(W^J, P_M\right)\left(Z_i^J\right)^{-\theta-1} X_i^J}{\zeta_i^J}. \tag{A.23}$$

Peretto (2008: Proposition 1) shows that when $\theta(\epsilon-1)<1$, the firm is always at the interior solution where $\zeta_i^J \alpha K^J = W^J$, and this in turn ensures that the equilibrium is *symmetric*, i.e. any producer of good X_i^J with $\forall i \in [0, N^J]$ follows the same optimality rules described above. Under symmetry, we have $\zeta_i^J \alpha K^J = W^J$ for each $i \in [0, N^J]$, that is,

$$\zeta_i^J = \frac{W^J}{\alpha K^J} = \frac{W^J}{\alpha \int_0^{N^J} \frac{1}{N^J} Z_i^J di} \text{ for each } i \in [0, N^J]. \tag{A.24}$$

Derivation of (18)-(19). Given (A.16), the conditional factor demands for $L_{X_i}^J$ and M_i^J of each firm are

$$L_{X_i}^J = \phi + \frac{\partial C_X^J(W^J, P_M)}{\partial W^J} (Z_i^J)^{-\theta} X_i^J = \phi + (1-\sigma) \frac{C_X^J(W^J, P_M)}{W^J} (Z_i^J)^{-\theta} X_i^J, \tag{A.25}$$

$$M_i^J = \frac{\partial C_X^J(W^J, P_M)}{\partial P_M} (Z_i^J)^{-\theta} X_i^J = \sigma \frac{C_X^J(W^J, P_M)}{P_M} (Z_i^J)^{-\theta} X_i^J. \tag{A.26}$$

Substituting $C_X^J(W^J, P_M) = \frac{\epsilon-1}{\epsilon} (Z_i^J)^{\theta} P_i^J$ from (A.22), equations (A.25)-(A.26) imply $W^J L_{X_i}^J = W^J \phi + (1-\sigma) \frac{\epsilon-1}{\epsilon} P_i^J X_i^J$ and $P_M M_i^J = \sigma \frac{\epsilon-1}{\epsilon} P_i^J X_i^J$. Integrating across varieties in both these expressions yields (18)-(19).

Derivation of (20). From (A.24), symmetry implies $Z_i^J = Z^J$ for each $i \in [0, N^J]$, and hence $K^J = \int_0^{N^J} \frac{1}{N^J} Z_i^J di = Z^J$, so that $\dot{K}^J / K^J = \dot{Z}^J / Z^J$. Given the accumulation rule $\dot{Z}^J = \alpha K^J L_{Z_i}^J$ we have $\dot{K}^J / K^J = \dot{Z}^J / Z^J = \alpha L_{Z_i}^J$. Since $L_{Z_i}^J = L_Z^J / N^J$, where L_Z^J is aggregate labor devoted to R&D projects in country J, we have

$$\dot{K}^J / K^J = \dot{Z}^J / Z^J = \alpha L_Z^J / N^J, \tag{A.27}$$

from which (20).

Derivation of (21) and (22). Multiplying by the number of entrants the first and last terms of (12), we have

$$\left(\dot{N}^J + \delta N^J\right) V_i^J = W^J L_N^J. \tag{A.28}$$

By symmetry, the value of production of any existing firm in instant t equals $P_i^J X_i^J = Y^J / N^J$, and the free-entry condition $V_i^J = \beta P_i^J X_i^J$ can be re-written as in (21). Plugging (21) in (A.28) and solving for $\dot{N}^J$ yields equation (22) in the text.

Derivation of (23). Denote the rate of return to vertical innovations as r_Z^J. From the monopolist problem, time-differentiate (A.24) and substitute (A.23) to eliminate $\dot{\zeta}_i^J / \dot{\zeta}_i^J$, obtaining

$$r_Z^J = \frac{\dot{W}^J}{W^J} - \alpha \frac{L_Z^J}{N^J} + \alpha \frac{\theta C_X^J(W^J, P_M) (Z_i^J)^{-\theta} X_i^J}{W^J} - \delta. \tag{A.29}$$

Next substitute $P_i^J X_i^J \left(\frac{\epsilon-1}{\epsilon}\right) = C_X^J(W^J, P_M) (Z_i^J)^{-\theta} X_i^J$ from (A.22), obtaining

$$r_Z^J = \frac{\dot{W}^J}{W^J} - \alpha \frac{L_Z^J}{N^J} + \alpha\theta \left(\frac{\epsilon-1}{\epsilon}\right) \frac{P_i^J X_i^J}{W^J} - \delta.$$

Substituting $P_i^J X_i^J = Y^J / N^J$ in the above expression and rearranging terms, we obtain (23).

Derivation of (24). Denote the rate of return to horizontal innovations as r_N^J. Time-differentiating V_i^J in (11) and in (21), yields

$$\frac{\dot{V}_i^J}{V_i^J} = r_N^J + \delta - \frac{\Pi_{X_i}^J}{V_i^J} \text{ and } \frac{\dot{V}_i^J}{V_i^J} = \frac{\dot{Y}^J}{Y^J} - \frac{\dot{N}^J}{N^J},$$

respectively. Combining the above expressions and solving for r^J gives

$$r_N^J = \frac{\dot{Y}^J}{Y^J} - \frac{\dot{N}^J}{N^J} + \frac{\Pi_{X_i}^J(t)}{V_i^J(t)} - \delta,$$

where we can substitute $\Pi_{X_i}^J(t) = \frac{1}{\epsilon}\frac{Y^J}{N^J} - W^J\phi - W^J L_{Z_i}^J$ from (A.17)-(A.22), and $V_i^J = \beta Y^J/N^J$ from (21) to obtain (24).

Derivation of (25)-(27). In the resource-processing sector of country H, the cost-minimizing conditions over L_M and R yield $\frac{W^H}{p} = \frac{\varsigma}{1-\varsigma}\left(\frac{R}{L_M}\right)^{\frac{1}{\tau}}$. The associated cost function is

$$C_M\left(W^H, p\right) = \left[(\varsigma)^\tau \left(W^H\right)^{1-\tau} + (1-\varsigma)^\tau p^{1-\tau}\right]^{\frac{1}{1-\tau}}, \tag{A.30}$$

and the conditional factor demands for raw resource and labor read

$$\begin{aligned} pR &= \frac{\partial C_M\left(W^H,p\right)}{\partial p}\frac{p}{C_M\left(W^H,p\right)}P_M M = S_M^R\left(W^H,p\right)\cdot P_M M, \\ W^H L_M &= \frac{\partial C_M\left(W^H,p\right)}{\partial W^H}\frac{W^H}{C_M\left(W^H,p\right)}P_M M = S_M^L\left(W^H,p\right)\cdot P_M M, \end{aligned}$$

where we have defined the elasticities of $C_M\left(W^H,p\right)$ to resource price and wage as $S_M^R\left(W^H,p\right)$ and $S_M^L\left(p,W^H\right)$, respectively. Recalling that $S_M^L\left(W^H,p\right) = 1 - S_M^R\left(W^H,p\right)$, the above expressions yield (25)-(26). Log-differentiating (13) we have (27).

Derivation of (28). Recalling that the global demand for the intermediate is $M = M^H + M^F$, eq.(18) implies $P_M M = \sigma\frac{\epsilon-1}{\epsilon}\left(Y^H + Y^F\right)$. Substituting this equation in (26), and imposing the market-clearing condition $R = \Omega$, we obtain (28).

Derivation of (29)-(30). Since $(1-\xi)$ is the share of expenditures on imported goods in both countries, the balanced trade condition (14) can be re-written as $P_M M^F = (1-\xi)E^H - (1-\xi)E^F$. Substituting $P_M M^F = \sigma\frac{\epsilon-1}{\epsilon}Y^F$ from (18), we have

$$\sigma\frac{\epsilon-1}{\epsilon}Y^F = (1-\xi)E^H - (1-\xi)E^F. \tag{A.31}$$

>From (15), substitute $Y^F = \xi E^F + (1-\xi)E^H$ in (A.31) to obtain

$$E^F/E^H = \frac{1-\sigma\frac{\epsilon-1}{\epsilon}}{1+\sigma\frac{\epsilon-1}{\epsilon}\frac{\xi}{1-\xi}}, \tag{A.32}$$

which is the first expression in (29). Substituting this result back in (A.31) to eliminate E^F we have

$$E^H/Y^F = 1 + \sigma\frac{\epsilon-1}{\epsilon}\frac{\xi}{1-\xi}. \tag{A.33}$$

Combining (A.32) and (A.33) we obtain $E^F/Y^F = 1 - \sigma\frac{\epsilon-1}{\epsilon}$, which is the second expression in (30). From (15), we also have $Y^H = \xi E^H + (1-\xi) E^F$, where E^F can be substituted by (A.32) to obtain $E^H/Y^H = \left(1 + \sigma\frac{\epsilon-1}{\epsilon}\frac{\xi}{1-\xi}\right) / \left(1 + \sigma\frac{\epsilon-1}{\epsilon}\frac{2\xi-1}{1-\xi}\right)$, which is the first expression in (30). Combining this result with (A.33) yields $Y^F/Y^H = \left[1 + \sigma\frac{\epsilon-1}{\epsilon}\frac{2\xi-1}{1-\xi}\right]^{-1}$, which is the second expression in (29).

Derivation of (31)-(34). Using $\Pi_M = 0$ and the free-entry condition $N^H V_i^H = W^H \beta Y^H$ we rewrite the dynamic constraint (6) as $\frac{\dot{N}^H}{N^H} + \frac{\dot{V}_i^H}{V_i^H} = r^H + \frac{L^H}{\beta Y^H} + \frac{p\Omega}{N^H V_i^H} - \frac{E^H}{N^H V_i^H}$. We then use $\frac{\dot{V}_i^H}{V_i^H} = \frac{\dot{Y}^H}{Y^H} - \frac{\dot{N}^H}{N^H}$ from (11) to obtain

$$\frac{\dot{Y}^H}{Y^H} = r^H + \frac{W^H L^H}{\beta Y^H} + \frac{p\Omega}{\beta Y^H} - \frac{E^H}{\beta Y^H}.$$

Substituting $r^H = (\dot{E}^H/E^H) + \rho$ from (16), and since $\dot{E}^H/E^H = \dot{Y}^H/Y^H$ from (29), we have

$$\frac{E^H}{Y^H} = \beta\rho + \frac{W^H L^H}{Y^H} + \frac{p\Omega}{Y^H}. \tag{A.34}$$

Defining $\mu \equiv 1/\left(1 + \frac{2\xi-1}{1-\xi}\sigma\frac{\epsilon-1}{\epsilon}\right)$, tedious but straightforward algebra shows that the first equation in (29) reduces to $E^H/Y^H = 1 + \sigma\frac{\epsilon-1}{\epsilon}\mu$. Substituting this expression into (A.34) and setting $W^H = 1$, we obtain $1 + \sigma\frac{\epsilon-1}{\epsilon}\mu = \beta\rho + \left(L^H + p\Omega\right)/Y^H$, which we can solve for Y^H to obtain (31). Also notice that (33) follows immediately from the second equation in (29). We derive (32) as follows. Re-writing (28) as $p\Omega = S_M^R(1,p) \cdot \sigma\frac{\epsilon-1}{\epsilon}\left(Y^H + Y^F\right)/L^H$, and using (33) to eliminate Y^F, we have

$$p\Omega = S_M^R(1,p)\,\sigma\frac{\epsilon-1}{\epsilon}(1+\mu)\,Y^H.$$

Substituting this expression in (31) to eliminate $p\Omega$, and solving for Y^H, we have (32), where $f(p)$ is defined in (35). We derive (34) as follows. Starting from the dynamic constraint (7), and following the same steps as for the derivation of (A.34), we obtain

$$\frac{E^F}{Y^F} = \beta\left(\rho + \frac{W^F L^F}{\beta Y^F}\right), \tag{A.35}$$

where, from (30), we can substitute $E^F/Y^F = 1 - \frac{\epsilon-1}{\epsilon}\sigma$ to obtain (34).

Proof of Proposition 1. Consider the determination of Y_*^H and p_* as described in Figure 1. Equation (31) is the straight line $Y_{(1)}^H$ increasing in p. Equation (32) is the curve $Y_{(2)}^H$ whose slope depends on the elasticity of substitution τ. If $\tau = 1$, eq.(27) implies $S_M^R = 1 - \varsigma$, eq.(35) implies $f(p) = \frac{\epsilon-1}{\epsilon}\sigma\left[(1+\mu)(1-\varsigma) - \mu\right]$, and eq.(32) implies that $Y_{(2)}^H$ is a straight line independent of p. If $\tau < 1$, eq.(27) implies $\partial S_M^R(1,p)/\partial p > 0$, eq.(35) implies $\partial f(p)/\partial p > 0$, and eq.(32) implies that $Y_{(2)}^H$ is an increasing function of p. If $\tau > 1$, eq.(27) implies $\partial S_M^R(1,p)/\partial p < 0$, eq.(35) implies $\partial f(p)/\partial p < 0$, and eq.(32) implies that $Y_{(2)}^H$ is a decreasing function of p. Notice that, from (31) and (32), an increase in Ω raises the slope of $Y_{(1)}^H$ without affecting $Y_{(2)}^H$. As a consequence, we have the results described in Figure 1, lower graphs: an increase in Ω reduces the equilibrium resource price p_* regardless of the value of τ, whereas the associated variation in equilibrium production Y_*^H is determined by

$$\frac{dY_*^H}{d\Omega}\left\{< 0 \text{ if } \tau < 1; \quad = 0 \text{ if } \tau = 1; \quad > 0 \text{ if } \tau > 1\right\}. \tag{A.36}$$

Equations (32) and (34) respectively imply that

$$sign\left(dY_*^H/d\Omega\right) = sign\left(dY_*^F/d\Omega\right) = sign\left(dW_*^F/d\Omega\right). \tag{A.37}$$

Moreover, from (31), we have $\Omega p_* = Y_*^H\left[1-\beta\rho+\frac{\epsilon-1}{\epsilon}\sigma\mu\right]-1$, which implies $sign\left(d\left(\Omega p_*\right)/d\Omega\right) = sign\left(dY_*^H/d\Omega\right)$. Combining this result with (A.36) and (A.37), Proposition 1 is proved. ■

Derivation of (37)-(38). Since Y_*^H is constant from system (31)-(34), expenditure E^H is constant from (30) and the Keynes-Ramsey rules (16) imply $r^H = \rho$. Hence, setting $W^H = 1$ and $r_N^H = r_Z^H = \rho$ in (23)-(24), we obtain

$$\rho+\delta = \alpha\left[\frac{Y_*^H}{N^H}\theta\left(\frac{\epsilon-1}{\epsilon}\right) - \frac{L_Z^H}{N^H}\right], \tag{A.38}$$

$$\rho+\delta = \frac{1}{\beta}\left[\frac{1}{\epsilon} - \frac{N^H}{Y_*^H}\left(\phi + L_{Z_i}^H\right)\right] - \frac{\dot{N}^H}{N^H}, \tag{A.39}$$

respectively. Substituting $L_Z^H/N^H = \alpha^{-1}\left(\dot{Z}^H/Z^H\right)$ from (20) in (A.38) yields

$$\frac{\dot{Z}^H}{Z^H} = \alpha\frac{Y_*^H}{N^H}\theta\left(\frac{\epsilon-1}{\epsilon}\right) - (\rho+\delta). \tag{A.40}$$

Result (A.40) implies that $\dot{Z}^H > 0$ if $N^H(t) < \bar{N}^H \equiv \frac{\alpha\theta}{\rho+\delta}\frac{\epsilon-1}{\epsilon}Y_*^H$, and $\dot{Z}^H = 0$ if $N^H(t) \geq \bar{N}^H$, which proves (37). Plugging $L_{Z_i}^H = L_Z^H/N^H$ and $\alpha^{-1}\left(\dot{Z}^H/Z^H\right)$ in (A.39), and recalling the two cases $N^H(t) < \bar{N}^H$ and $N^H(t) \geq \bar{N}^H$ established in (37), we obtain (38).

Derivation of (39)-(40)-(41). Consider a path where $N^H(t) < \bar{N}^H$ $\forall t$. Collecting the constant terms in $\nu \equiv \frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\rho+\delta)$, the first equation in (38) becomes

$$\frac{\dot{N}^H(t)}{N^H(t)} = \nu\left(1 - \frac{N^H(t)}{N_{ss}^H}\right), \tag{A.41}$$

which can be directly integrated to obtain the solution (39). Setting $\dot{N}^H = 0$ yields the steady-state N_{ss}^H defined in (40). Letting $t \to \infty$ in (39) proves that $\lim_{t\to\infty} N^H(t) = N_{ss}^H$. Substituting (39) in (37) with $N^H(t) < \bar{N}^H$, we have

$$\frac{\dot{Z}^H(t)}{Z^H(t)} = \left[1 - \left(1 - \frac{N_{ss}^H}{N_0^H}\right)e^{-\nu t}\right]\frac{Y_*^H}{N_{ss}^H}\alpha\theta\frac{\epsilon-1}{\epsilon} - (\rho+\delta).$$

Letting $t \to \infty$ yields (41).

Proof of Proposition 2. See the main text above the Proposition.

Derivation of (42)-(45). Using (38) to eliminate $\dot{N}^H/N^H$ from (22) we obtain

$$\left[\frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\rho+\delta)\right]\left[1 - \frac{W_*^H}{Y_*^H}\frac{\frac{1}{\beta}\left(\phi - \frac{\rho+\delta}{\alpha}\right)}{\frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\rho+\delta)}N^H(t)\right] = \frac{W^H L_N^H}{\beta Y_*^H} - \delta$$

which can be solved for L_N^H with $W_*^H = 1$ to obtain (42). Using (37) to substitute $\dot{Z}^H/Z^H$ in (20) we obtain (43). Setting $W^H = 1$ in (19) we obtain (44). Substituting (42)-(44) in the market clearing condition $L_M = L^H - L_X^H - L_Z^H - L_N^H$ we obtain (45). Recalling that $\lim_{t\to\infty} N^H(t) = N_{ss}^H$, where N_{ss}^H is given by (40), equations (42) and (43) imply that

$$\lim_{t\to\infty} L_N^H(t) = \delta\beta Y_*^H \text{ and } \lim_{t\to\infty} L_Z^H(t) = \frac{\tilde{\alpha}}{\epsilon} Y_*^H, \tag{A.42}$$

where $\tilde{\alpha} \equiv \alpha\phi\theta(\epsilon-1) - (\rho+\delta) + \beta\epsilon(\rho+\delta)^2 > 0$. Results (A.42) imply that, in response to a resource boom, the long-run values of L_N^H and L_Z^H react in the same direction as Y_*^H, converging to a higher (lower) steady state when $\tau > 1$ ($\tau < 1$).

Derivation of (46)-(47). By symmetry across varieties, (1) yields

$$\log u^H = \xi \log\left(N^H\right)^{\frac{\epsilon}{\epsilon-1}} \left(X_i^{Hh}/L^H\right) + (1-\xi)\log\left(N^F\right)^{\frac{\epsilon}{\epsilon-1}} \left(X_i^{Fh}/L^H\right). \tag{A.43}$$

(A.11) and (A.12) yield X_i^{Hh} and X_i^{Fh} as constant fractions of the respective manufacturing production levels. Setting $\chi^H \equiv \left[1 + \frac{1-\xi}{\xi}\cdot\frac{E^F}{E^H}\right]^{-1}$ and $\chi^F \equiv \left[1 + \frac{1-\xi}{\xi}\cdot\frac{E^H}{E^F}\right]^{-1}$, we write $X_i^{Hh} = \chi^H X_i^H$ and $X_i^{Fh} = \chi^F X_i^F$. Next, we note that $Y^J = N^J P_i^J X_i^J$ implies $N^J X_i^J = Y_*^J / P_i^J$. Consequently, we write

$$X_i^{Hh} = \chi^H \frac{Y_*^H}{N^H P_i^H} \text{ and } X_i^{Fh} = \chi^F \frac{Y_*^F}{N^F P_i^F}.$$

Using (A.22) to eliminate P_i^H and P_i^F, we obtain

$$X_i^{Hh} = \left(\bar{\chi}^H/N^H\right)\left(Z_i^H\right)^\theta \text{ and } X_i^{Fh} = \left(\bar{\chi}^F/N^F\right)\left(Z_i^F\right)^\theta, \tag{A.44}$$

where $\bar{\chi}^J \equiv \chi^J \frac{\epsilon-1}{\epsilon}\frac{Y_*^J}{C_X^J(W_*^J, P_M)}$. Substituting (A.44) in (A.43), and recalling that $Z_i^J = Z^J$ in a symmetric equilibrium, we obtain (46), where

$$\Upsilon_*^H \equiv \frac{\epsilon-1}{\epsilon}\left[\frac{\chi^H \frac{Y_*^H}{L^H}}{C_X^H\left(1, C_M\left(1, p_*\right)\right)}\right]^\xi \left[\frac{\chi^F \frac{Y_*^F}{L^F}}{C_X^F\left(W_*^F, C_M\left(1, p_*\right)\right)}\frac{L^F}{L^H}\right]^{1-\xi}.$$

We follow similar steps to derive (47). The difference is that we have $X_i^{Hf} = \left(1-\chi^H\right) X_i^H$, $X_i^{Ff} = \left(1-\chi^F\right) X_i^F$ and

$$\Upsilon_*^F \equiv \frac{\epsilon-1}{\epsilon}\left[\frac{\left(1-\chi^H\right)\frac{Y_*^H}{L^H}}{C_X^H\left(1, C_M\left(1, p_*\right)\right)}\frac{L^H}{L^F}\right]^\xi \left[\frac{\left(1-\chi^F\right)\frac{Y_*^F}{L^F}}{C_X^F\left(W_*^F, C_M\left(1, p_*\right)\right)}\right]^{1-\xi}.$$

Proof of Proposition 3. Result (50) is derived as follows. Defining $\Delta^J \equiv \left(N_{ss}^J/N_0^J\right) - 1$, we write (39) as

$$N^J(t) = N_0^J \frac{1+\Delta^J}{1+\Delta^J e^{-\nu\cdot t}}. \tag{A.45}$$

Denoting by $g^J(t)$ the growth rate of $\left(Z^J(t)\right)^{\theta}$, we have

$$\begin{aligned} g^J(t) &= \theta\frac{\dot{Z}^J(t)}{Z^J(t)} = \theta\left[\frac{Y_*^J}{W_*^J N^J(t)}\alpha\theta\left(\frac{\epsilon-1}{\epsilon}\right)-(\rho+\delta)\right], && \text{(A.46)} \\ g_{ss}^J &\equiv \lim_{t\to\infty} g^J(t) = \theta\left[\frac{Y_*^J}{W_*^J N_{ss}^J}\alpha\theta\left(\frac{\epsilon-1}{\epsilon}\right)-(\rho+\delta)\right], && \text{(A.47)} \end{aligned}$$

and we can write the time path

$$\theta\log Z^J(t) = \theta\log Z^J(0) + g_{ss}^J\cdot t + \int_0^t \left[g^J(s)-g_{ss}^J\right]ds. \qquad \text{(A.48)}$$

>From (A.46)-(A.47), the term in the integral can be written as

$$g^J(s) - g_{ss}^J = \gamma^J\left(\frac{N_{ss}^J}{N^J(t)}-1\right), \qquad \text{(A.49)}$$

where we have defined $\gamma^J \equiv \theta\cdot\alpha\theta\left(\frac{\epsilon-1}{\epsilon}\right)\frac{Y_*^J}{W_*^J N_{ss}^J}$. Since $\left(N_{ss}^J/N^J(t)\right)-1 = \Delta^J e^{-\nu\cdot t}$, integration of (A.49) yields

$$\int_0^t \left[g^J(s)-g_{ss}^J\right]ds = \gamma^J\int_0^t \Delta^J e^{-\nu\cdot s}ds = \frac{\gamma^J\Delta^J}{\nu^J}\left(1-e^{-\nu\cdot t}\right). \qquad \text{(A.50)}$$

Substituting (A.50) into (A.48) we obtain

$$\theta\log Z^J(t) = \theta\log Z^J(0) + g_{ss}^J\cdot t + \frac{\gamma^J\Delta^J}{\nu^J}\left(1-e^{-\nu\cdot t}\right). \qquad \text{(A.51)}$$

>From (A.45) and (A.51), it follows that the log of TFP levels evolves according to

$$\log T^J(t) = \log T_0^J + g_{ss}^J\cdot t + \left[\frac{\gamma^J}{\nu^J}\Delta^J\left(1-e^{-\nu\cdot t}\right)+\frac{1}{\epsilon-1}\log\frac{1+\Delta^J}{1+\Delta^J e^{-\nu\cdot t}}\right].$$

without loss of generality, we can approximate

$$\log\frac{1+\Delta^J}{1+\Delta^J e^{-\nu\cdot t}} \simeq \Delta^J\left(1-e^{-\nu\cdot t}\right)$$

and thus write (50). ■

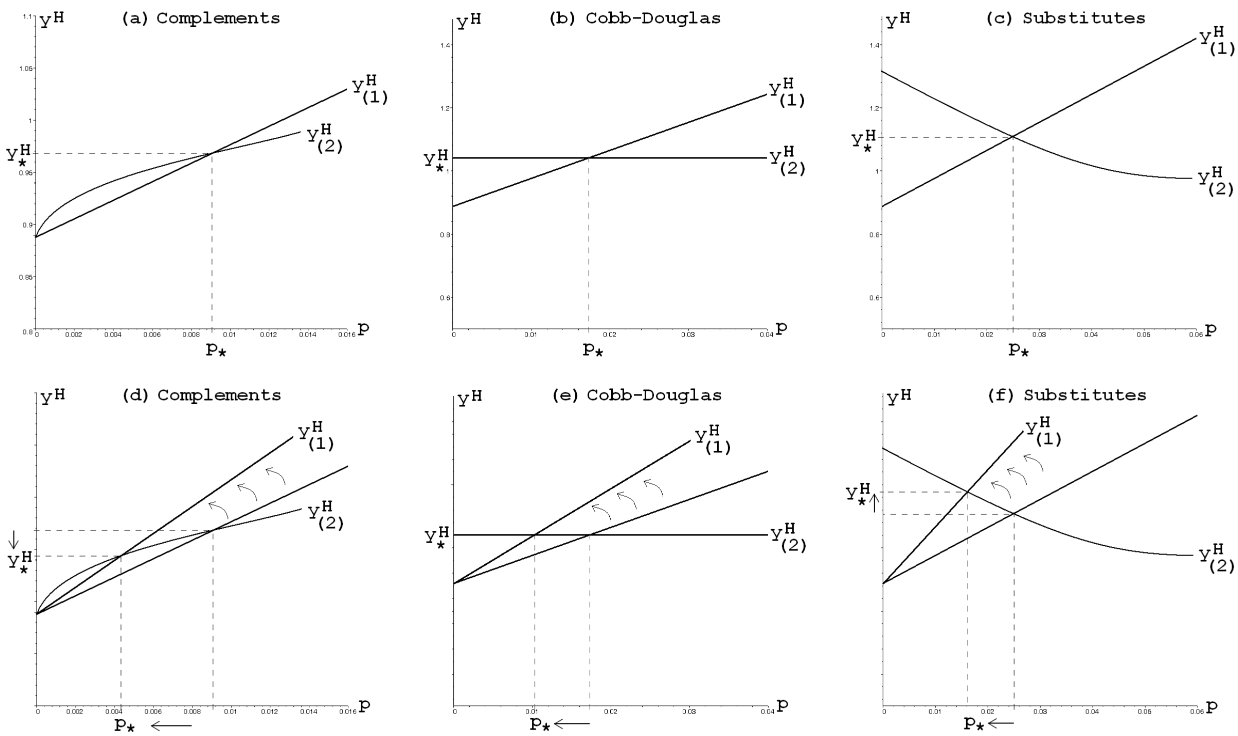

Figure 1: Upper graphs: equilibrium determination of Home expenditure and resource price when (a) $\tau < 1$, (b) $\tau = 1$, (c) $\tau > 1$. Lower graphs: effects of a resource boom when (c) $\tau < 1$, (d) $\tau = 1$, (e) $\tau > 1$.

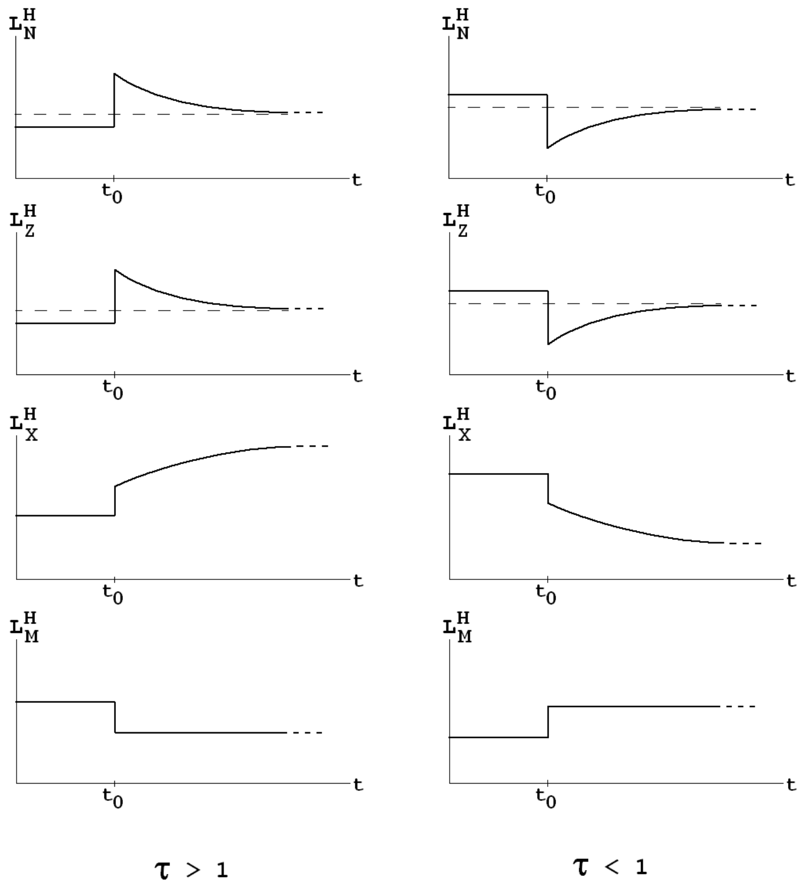

Figure 2: Transitional dynamics of employment levels in Home sectors after the resource boom when $\tau > 1$ (left graphs) and $\tau < 1$ (right graphs).