Interdependence of Trade Policies in General Equilibrium*

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Abstract

This paper sheds light on the nature of the interdependencies among various trade policy instruments. Adopting a rich general equilibrium, multi-industry model of trade, we find that (i) Sectoral import tariffs are complementary (i.e., restricting tariffs in a subset of sectors lowers the optimal tariff in unrestricted sectors), (ii) Import policy is only an imperfect substitute for export policy, and (iii) Non-revenue trade barriers (such as import bans or inefficient customs regulation) can be optimal in some sectors, serving as an imperfect substitute for tariffs. These policy interdependencies play an important role in the optimal design of trade agreements and provide a novel perspective on the WTO's ban on export subsidies. Fitting our model to trade data from the United States and China, we show that these policy interdependencies are also quantitatively significant.

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1 Introduction

Governments can use various policy instruments to influence international trade flows in favor of their economic objectives. These instruments, which include border and domestic taxes and subsidies, and a multitude of less transparent trade barriers such as inefficient customs regulations, are potentially interdependent. For example, in trade agreements, the benefits of liberalization in certain areas may be offset by the governments' policy reactions in other areas (Horn, Maggi, and Staiger, 2010).[1] Moreover, as emphasized by Goldberg and Pavcnik (2016), a true measure of trade policy must take into account the prevalence of non-tariff measures that have been used by the governments as an imperfect substitute for tariffs. Therefore, understanding trade policy interdependencies is critical in analyzing the policy reactions of the governments to changes in the political economy environment or ratification of new trade deals that constrain the governments' economic policy space.

Despite their importance, trade policy interdependencies are largely overlooked in the existing literature. A large class of quantitative trade models focus on only one policy instrument: non-revenue or iceberg trade barriers. The literature on optimal policy, meanwhile, considers multiple policy instruments but abstracts from policy interdependencies for two main reasons. First, the focus of this literature is on "optimal" policy rather than the tradeoffs that policymakers face outside the optimum. This focus is despite the fact that in practice, as a result of trade negotiations, political pressures, etc, applied policies most often deviate from any given notion of unilateral "optimum". Second, optimal policy analysis across multiple sectors is usually conducted in partial equilibrium setups that preclude intersectoral policy linkages.

Our objective in this paper is to take a step toward closing this gap in the

[1] In a model of incomplete trade agreements, Horn et al. (2010) show that governments will have an incentive to use domestic subsidies in response to negotiated tariff cuts. The increase in domestic subsidies after entering a trade agreement tends to partially offset the benefits from negotiated trade liberalization.

literature. We characterize the interdependence of various trade policies in a multi-sector general equilibrium model in which policies in various sectors are related. In this process, we analyze the *constrained* optimal policy when the government's policy space is constrained by factors such as international trade agreements.

We work within a standard class of gravity trade models, which provide a rich general equilibrium environment to study policy interdependence. The gravity models, in their various incarnations, have become the dominant framework to study international economic problems. However, despite their popularity and outstanding empirical success, these models have rarely been used to study optimal sectoral trade policy.[2]

As an intermediate step towards analyzing trade policy interdependence, we provide an analytical characterization of optimal trade policy in a large class of multiple-sector general equilibrium gravity models with intra-industry trade and heterogeneous trade elasticities across sectors. This is a necessary step, as we cannot appeal to existing theories for a solution. The most relevant characterization of optimal policy is Costinot, Donaldson, Vogel, and Werning (2015), who derive the optimal policy for a special case of the gravity framework where trade elasticities are uniform across sectors.[3] Given that trade elasticity heterogeneity is a key element in quantitative gravity models, we develop a novel methodology to solve for the optimal tax schedule in this more general setting.[4]

[2]Several authors, including Ossa (2014) and Caliendo and Parro (2014), have highlighted the inherent complexities of analyzing trade policy in multi-sector general equilibrium gravity models by adopting a computational approach. Beyond these quantitative studies, the analytics of optimal policy within multi-sector gravity models remain largely unknown.

[3]These models include Costinot et al. (2011), Chaney (2008), and Fieler (2011). We show that our methodology is flexible enough to be applied to even richer general equilibrium environments featuring input-output linkages or firm-heterogeneity.

[4]We find that the unconstrained optimal policy schedule includes zero non-revenue trade barriers, uniform tariffs, t^*, and a sector-specific export taxes, x_k^*, such that $(1+t^*)\left(1+x_k^*\right)=\frac{1}{\theta_k \lambda_{ff,k}}$, where θ_k is sector k's trade elasticity, and $\lambda_{ff,k}$ is the share of foreign country's expenditure on local varieties in sector k. This formula indicates that the sector-level trade elasticity has a first order effect on the optimal tax schedule.

We introduce three novel policy interdependence results, namely: *(i)* Import tariffs across sectors are complementary; *(ii)* Import policy is an imperfect substitute for export policy; *(iii)* Non-Revenue Trade Barriers (NRTBs), also known as wasteful trade barriers, may be optimal in the absence of more efficient trade policy instruments such as tariffs. As we describe below, these results have important implications about the design and the consequence of trade agreements.

To obtain a general intuition about our first result that *import tariffs across sectors are complementary*, note that an (exogenous) increase in the tariff of one sector reduces the relative wage of the foreign country by depressing the relative demand for foreign labor. Given the mobility of labor across sectors, the reduction in the foreign wage implies an improved terms of trade and higher import volumes in all other sectors in the home country, which in turn increases home country's marginal value of tariffs in those sectors.

Our second result about the interdependence of import and export policies is akin to—but distinct from—the Lerner symmetry theorem. We find that import policy is only an imperfect substitute for export policy—i.e., the equilibrium obtained by optimal import tariffs may be exactly replicated by a set of export policies; but no set of import tariffs could replicate the equilibrium under the optimal export taxes. Therefore, a government cannot achieve the optimal trading equilibrium using import policies alone. In other words, export policy is more *potent* than import policy as an instrument to manipulate a country's terms of trade.

The above policy interdependence results provide novel predictions about the choice of applied tariffs under an incomplete agreement.[5] In particular, the tariff complementarity result implies that negotiated tariff cuts in a subset of sectors lead to unilateral (i.e., voluntary) tariff cuts in the un-

[5] A trade agreement is incomplete if it regulates only a subset of relevant policies or contingencies. It may be optimal to leave a trade agreement incomplete due to the *cost of writing* a complete agreement (as in Horn, Maggi, and Staiger, 2010) or due to the *cost of implementing* such an agreement (as in Beshkar and Bond 2017).

bound and loosely bound sectors.[6] Moreover, the imperfect substitution between import and export policies implies that a negotiated ban on export subsidies—even without any negotiation on tariff cuts—will reduce unilateral import tariffs *and* the effective level of protection in a country.[7]

Our finding that import and export policies are imperfect substitutes offers a novel perspective on the WTO's ban on export subsidies. From the standpoint of standard terms-of-trade models, a ban on export subsidies is puzzling since export subsides have positive—rather than negative—externalities. By contrast, given our finding that in a multisector general equilibrium framework export policy is more potent than import policy in manipulating terms of trade, a ban on export subsidies, even without any restrictions on import tariffs, could reduce the overall level of protection administered by each country. To see the novelty of this result, note that in a standard two-sector general equilibrium model, a ban on export policy will have no impact on the effective level of protection.[8]

Our third interdependence result is related to *Non-Revenue Trade Barriers* (NRTBs) as an alternative protectionist measure. NRTBs include measures such as iceberg transport costs, import bans and inefficient customs regulations (i.e., red tapes at the border) that discourage imports but do not generate revenues. In response to tariff cuts, many countries have opted for non-tariff barriers, most of which do not generate any revenues for the gov-

[6]An unbound sector in a country is one in which no tariff binding is negotiated and, thus, the government is free to choose its tariff unilaterally. A loosely-bound sector is a sector in which a tariff binding is negotiated but the binding is higher than the government's unilateral choice of applied tariff—a phenomenon known as *tariff overhang*. Beshkar et al. (2015); Beshkar and Bond (2017) provide evidence on the prevalence of sectors with significant tariff overhang.

[7]These predictions are in line with the observation that developing countries cut their tariffs unilaterally in sectors where they had high negotiated bindings under the WTO (Martin and Ng 2004). Baldwin 2010 attributes these unilateral tariff liberalizations to the fragmentation of the production processes. Our theoretical finding that tariffs are complementary suggest that unilateral tariff liberalization in unbound or loosely bound sectors could also be the consequence of negotiated tariff cuts in other sectors.

[8]That is because due to Lerner's Symmetry, in a model with one import and one export sector, import and export policies are perfectly substitutable and, thus, no liberalization may be achieved by restricting only one of the instruments.

ernments. As emphasized by Goldberg and Pavcnik (2016), a true measure of trade policy must take into account the prevalence of non-tariff measures, especially after the implementation of negotiated tariff cuts. Therefore, to analyze the impact of trade policy, it is imperative to understand the incentives of the governments in choosing NRTBs, the likely pattern of NRTBs chosen by the governments, and their welfare implications.

From the perspective of partial-equilibrium trade models, a rise in NRTBs is hard to explain. That is because such measures reduce trade without compensating the resulting consumption losses with a better terms of trade. Under a general equilibrium framework, however, we show that in the absence of revenue-generating measures, NRTBs could improve a country's welfare by improving its terms of trade.

The optimality of NRTBs under a constrained policy space follows because restricting imports in one sector improves a country's terms of trade in *all other sectors* by depressing foreign wages. Therefore, if the consumption loss due to import restriction in a sector is sufficiently small, imposing an NRTB in that sector could be optimal. We show that this condition is satisfied in relatively homogenous sectors where imported varieties could be easily substituted with domestic counterparts. In the case of the US, trade data suggests that such sectors include wheat, rice, dairy, and apparel.

We conduct a calibration exercise to quantify the empirical relevance of our findings, including the size of the optimal policy and the relative importance of trade elasticities and comparative advantage as determinants of optimal policy.[9] Assuming that export policies are banned, we find that the optimal import tariff for the United States and China to be around 68% and 56%, respectively. Moreover, we find that 99% of the sectoral variation in optimal export taxes is driven by the variation in sector-level trade elasticities and only a mere 1% is driven by forces of comparative advantage. Regrading the size of the underlying welfare effects, we calculate that the

[9]A practical advantage our analytical formula is to reduce computation time significantly. Using our formula, we can solve for the optimal tax schedule 27 times faster than standard methods.

imposition of unilaterally optimal trade taxes by China can raise its real income per capita by 2.46 percentage points. However, the welfare increase would be 0.20 percentage points lower if export policies were unavailable. This last finding highlights the imperfect substitutability of import and export policies.

To illustrate the role of policy interdependencies, we conduct two counterfactual analyses that correspond to some hypothetical *incomplete* trade agreements. First, to highlight the impact of the WTO's ban on export subsidies, we consider an agreement under which export subsidies are banned but governments are allowed to choose their import tariffs unilaterally. We calculate that, starting from a noncooperative equilibrium, the ban will reduce the optimal import tariffs of the United States by about 44 percentage points. We then consider a *more complete* agreement under which, in addition to a ban on all export subsidies, tariffs are eliminated in half of the traded sectors. For the United States, we compute that this agreement will lead to an additional 43 percentage point reduction in the optimal tariffs in the unrestricted sectors.

The paper is organized as follows. After discussing the related literature in the next subsection, in Section 2, we lay down the gravity framework that we use throughout the paper. In Section 3, we derive the optimal tax/subsidy schedule. We then analyze the interdependence of trade policies in Section 4 and the optimality of NRTBs in absence of tariffs in Section 4.3. Quantitative analyses is presented in Section 5. In Section 6 we provide concluding remarks including a discussion on the implications of policy interdependencies for trade negotiations, and the cause of the optimality of uniform tariffs within the models we consider. In the Appendices, in addition to proofs, we provide extensions of our optimal policy analysis under input-output linkages and monopolistic competition.

1.1 Related Literature

Optimal Trade Policy

Our results regarding the optimal schedule of trade taxes cover the previous general equilibrium characterizations of the optimal trade policy including Costinot et al. 2015, Opp 2010, and Itoh and Kiyono 1987. While Opp 2010 focuses on import tariffs and Itoh and Kiyono 1987 focus on export subsidies, Costinot et al. 2015 consider the simultaneous choice of import and export policies and show the optimality of uniform import tariffs for the case where trade elasticities are the same across sectors and preferences are additively separable. We show that these results continue to hold in an environment with heterogenous trade elasticities across sectors and a general (not necessary separable) preference structure.

The idea that optimal trade policy for a product should depend on the *elasticity* of its supply and demand was proposed by Bickerdike (1906) and was later popularized by others including Kahn (1947), who calculated the exact formula for optimal import tariff to be equal to the inverse of the foreign export supply elasticity. This approach came under criticism due to its disregard for general equilibrium effects (Graaff 1949; Horwell and Pearce 1970; Bond 1990). Nevertheless, those criticisms were mostly suggestive and did not provide a practical framework to evaluate general-equilibrium effects of trade policy. The subsequent literature, perhaps for practical reasons, adopted Bickerdike's "elasticity approach" to study the variation in sectoral trade policies (e.g., Grossman and Helpman 1995; Broda et al. 2008; Bagwell and Staiger 2011; and Beshkar et al. 2015).[10] In this paper, by providing an analytical characterization of optimal trade policy (both constrained and unconstrained), we offer a practical way to analyze trade policy in general equilibrium.

We are unaware of any previous work that views NRTBs as a beggar-

[10] The existing general equilibrium analyses of trade policy are either conducted for a small open economy (as in the tariff reform literature cited below), or a two-sector economy with only one import good and one export good (e.g., Bagwell and Staiger 1999, Limão and Panagariya 2007).

thy-neighbor policy. The existing studies of non-tariff barriers as policy variables–e.g., Berry, Levinsohn, and Pakes (1999), Harrigan and Barrows (2009), and Maggi, Mrázová, and Neary (2017)—view them implicitly as an instrument to transfer wealth to interest groups without generating any welfare gains at the national level. Similar to our study of NRTBs, Maggi, Mrázová, and Neary (2017) analyze the use of wasteful trade barriers when the governments' policy space is constrained by a trade agreement. They show that if tariff commitments could not be fully contingent on political realizations, the extent of tariff liberalization is limited by the need to prevent such wasteful behavior. Our framework offers a complementary perspective on NRTBs as instruments that could be potentially used to improve a country's terms of trade in expense of foreign countries.

A growing literature, including Demidova and Rodríguez-Clare (2009) Felbermayr, Jung, and Larch (2013), Haaland and Venables (2016), Costinot, Rodríguez-Clare, and Werning (2016), and Caliendo, Feenstra, Romalis, and Taylor (2015)analyzes trade policy under the monopolistically competitive framework of Melitz (2003). All of these papers focus on models with a *single tradable sector* and, thus, their results are not readily comparable to our findings regarding the optimal policy across *multiple* sectors. A partial exception is Costinot, Rodríguez-Clare, and Werning (2016) who study *firm*-specific policies and find that within the same sector, optimal firm-specific tariffs are increasing in the productivity of the foreign firms.

Our theory contributes to a growing literature that attempts to quantify the trade policy equilibrium of *optimizing* governments (Perroni and Whalley, 2000; Ossa, 2011, 2012, 2014). This literature, which is aptly discussed by Ossa (2016) and Costinot and Rodriguez-Clare (2013), uses numerical optimization to find the tariff choice of optimizing governments. Numerical optimization is often plagued with the curse of dimensionality when many sectors are involved. Applying our theory to trade data, we show that our analytical formulas facilitate the computational task in such cases. Moreover, we take a first step towards highlighting the empirical significance of cross-price elasticity effects in the design of optimal policy.

Interdependence

As noted above, the existing literature is mostly silent about trade policy interdependencies due to its focus on "optimal" policy–rather than the tradeoffs that policymakers face outside the optimum– and partial equilibrium, which precludes interrelations across sectors. Partial exceptions include the literatures on tariff complementarity in Free Trade Areas and the Piecemeal Tariff Reforms, which we now discuss.

There is a literature on tariff complementarity in Free Trade Areas (FTA). While we find that tariffs across sectors within a country are complementary, Richardson (1993), Bond, Riezman, and Syropoulos (2004) and Ornelas (2005) find that for members of a Free Trade Area (FTA), internal and external tariffs are complementary. In particular, they find that as a response to tariff cuts within an FTA, the member countries will voluntarily reduce their tariffs on imports from non-members. Similarly, in a North-South model, Zissimos (2009) considers tariff complementarities across countries within a region that compete for imports from the rest of the world.

The theory of piecemeal tariff reform (Hatta 1977; Fukushima 1979; Anderson and Neary 1992, 2007; Ju and Krishna 2000) is another strand of the literature that touches on the issue of policy interdependence. This literature is primarily concerned with welfare-enhancing tariff reforms that are revenue-neutral (or revenue-enhancing) in a *small open economy*. A general finding of the piecemeal reform literature is that compressing the variation of existing tariffs in developing countries—by reducing the highest tariff rates and increasing the lowest ones—could increase welfare without decreasing revenues. Although we focus on an entirely different problem in this paper, our finding about the optimality of uniform tariffs resonates with this literature's recommendation for tariff reforms.

As in this paper, Bagwell and Lee (2015) provide a perspective on the WTO's ban on export subsidies. Within a heterogenous-firm model, Bagwell and Lee (2015) show that if import tariffs (as well as transportation costs) are very low, then an export subsidy may benefit a country at the expense of its trading partners. Their finding suggests that a ban on export subsidies is useful only after substantial liberalizations have been reached through previous negotiations. By contrast, our analysis suggests that a ban on export subsidies is useful even without any restrictions on import tariffs.

Another related literature studies issue linkages in international relations. This literature considers various conditions under which there might be an interdependence between trade policies and non-trade policies—such as environmental policies (Ederington, 2001, 2002; Limão, 2005), production subsidies (Horn, Maggi, and Staiger, 2010), and intellectual property protection. These papers draw conclusions about whether these non-trade issues should be linked to trade agreements (see Maggi 2016 for a review).

2 The Economic Environment

The economy consists of two countries: h (Home) and f (Foreign). There are K asymmetric sectors indexed by k, where the number of sectors could be arbitrarily large and possibly infinite. We assume that markets are perfectly competitive and that labor is the only factor of production, with L_i denoting the labor endowment in country i and w_i denoting the labor wage.[11]

The utility of the representative consumer in country $i = h, f$ is given by

$$W_i = U_i(Q_{i,1}, ..., Q_{i,K}),$$

where $Q_{i,k}$, denotes aggregate consumption in sector k. Our demand structure is a step towards generality, as it does not require the *separability* assumption underlying many theories of optimal policy—e.g., Kahn (1947); Grossman and Helpman (1995); Costinot et al. (2015)). We also take a flexible stance with respect to intra-sector (or intra-product) trade. Instead of assuming homogeneous sectors and complete specialization (à la Ricardo), we impose the following structure on sector-level consumption, $Q_{i,k}$:

[11]The model could accommodate multiple factors of production. All forthcoming propositions carry over to a multi-factor setting where the share of each factor in production is uniform across sectors.

R1. *Within sectors, the import demand system is CES in that the price index of the aggregate consumable in sector k is given by*

$$P_{i,k} = \left(\sum_{j=h,f} A_{ji,k} \left[\tau_{ji,k} \left(1 + t_{ji,k}\right) \left(1 + x_{ji,k}\right) w_j \right]^{-\theta_k} \right)^{-\frac{1}{\theta_k}},$$

and the share of country i's expenditure on country j exports in sector k is given by a gravity equation:

$$\lambda_{ji,k} = \frac{A_{ji,k} \left[\tau_{ji,k} \left(1 + t_{ji,k}\right) \left(1 + x_{ji,k}\right) w_j \right]^{-\theta_k}}{\sum_{n=h,f} A_{ni,k} \left[\tau_{ni,k} \left(1 + t_{ni,k}\right) \left(1 + x_{ni,k}\right) w_n \right]^{-\theta_k}},$$

where θ_k denotes the trade elasticity; $\tau_{ji,k}$ denotes non-revenue (iceberg) trade barriers, $x_{ji,k}$ denotes export tax, and $t_{ji,k}$ denotes import tax all applied to exports from country j to i in sector k; and finally $A_{ji,k}$ is a function of only structural parameters.

The above structure flexibly accommodates both *within-sector* trade driven by forces of gravity and *across-sector* trade driven by comparative advantage (θ_k regulates the former, whereas $A_{ji,k}$ governs the latter). The *within-sector* import demand structure specified by R1 has deep theoretical roots (see Eaton and Kortum 2002 and Anderson and Van Wincoop 2003), but is not foundational to our analysis. When $\theta_k \to \infty$ each sector or product is sourced from the most efficient supplier, and our framework collapses into a Ricardian model with a general demand structure, $U_i(.)$ across sectors or products. In fact, we impose no restrictions on how the trade elasticities and efficiency levels vary across sectors.

The structure outlined by R1 accommodates three policy instruments: (i) revenue-generating ***import taxes***, $t_{ji,k}$, and (ii) revenue-generating ***export taxes***, $x_{ji,k}$, and (iii) ***non-revenue trade barriers*** (NRTBs), $\tau_{ji,k}$, which have been the main focus of the quantitive trade literature in the past decade (see Costinot and Rodriguez-Clare 2013). In our analysis, NRTBs can be either zero ($\tau_{ji,k} = 1$) or strictly positive ($\tau_{ji,k} > 1$). Additionally, we abstract from tax on domestic sales ($x_{ii,k} = t_{ii,k} = 0$) and domestic NRTBs ($\tau_{ii,k} = 1$).

With labor as the only factor of production, total income in country $i = h, f$ equals the sum of labor income, $w_i L_i$, plus the tax revenue collected across all sectors:

$$Y_i = w_i L_i + \sum_{k=1}^{K} t_{ji,i} X_{ji,k} + \sum_{k=1}^{K} \frac{x_{ij}}{1 + x_{ij}} X_{ij,k},$$

Given income and trade shares, one can calculate sector-specific trade values as $X_{ji,k} = \lambda_{ji,k} Y_{i,k}$, where $Y_{i,k}$ denotes total expenditure on sector k with $\sum_k Y_{i,k} = Y_i$. Hereafter, we use labor in the Home country as our numeraire, letting $w \equiv w_{f/} w_h$ denote the wage in Foreign relative to Home.

As an equilibrium condition, we restrict international trade to be balanced—i.e., Home's total imports from Foreign equal its total exports.

R2. *Trade is balanced:* $\sum_{k=1}^{K} X_{fh,k} = \sum_{k=1}^{K} X_{hf,k}$.

Finally, $A_{ji,k}$ which encompasses structural parameters such as efficiency levels, taste, and transport costs can vary freely across countries and sectors. However, we assume that $A_{ji,k}$ is invariant to policy. That is, tastes or productivity levels cannot be manipulated with trade policy instruments.

R3. $A_{ji,k}$ *is inavariant to policy.*

The above restriction resembles the linear cost assumption in Arkolakis et al. (2012). It rules out diminishing returns to scale (due to sector-specific factors of production) or increasing returns to scale (due to Marshallian externalities).

Despite restrictions R1-R3, our framework retains generality and nests an important class of canonical general equilibrium multi-sector trade models. For example, our framework nests a wide range of homothetic and non-homothetic multi-sector gravity models (e.g., Costinot et al. 2011 and Fieler 2011). It also nests Dornbusch, Fischer, and Samuelson's (1977) Ricardian model with homogeneous sectors as a special case. Additionally, as we show in Appendix G, our methodology could be applied in richer environments with input-output linkages and firm heterogeneity (e.g., Chaney 2008).

3 Unconstrained Optimal Trade Policy

As an intermediate step towards analyzing trade policy interdependence, this Section provides an analytical characterization of optimal trade policy without any constraints on the government's policy space. We then study policy interdependencies in the subsequent sections by characterizing the *constrained* optimum.

We focus on the Home country's optimal policy, taking non-revenue trade barriers in Foreign, $\tau_{hf,k}$, as given and setting the Foreign trade tax to zero. In this setup, we use the following notation: $\tau_k \equiv \tau_{fh,k}$ denotes non-revenue trade barriers at Home, $t_k \equiv t_{fh,k}$ denotes Home's import tariffs, and $x_k \equiv x_{hf,k}$ denotes Home's export tax, all in sector k. Given total income, Y_h, and the vector of sector-level price indexes, $\mathbf{P}_h = \{P_{h,k}\}$, Home's aggregate welfare is uniquely described by:

$$W_h = V_h(Y_h, \mathbf{P}_h).$$

In the above environment, Home's optimum policy schedule involves a vector of sector-level export taxes, $\boldsymbol{x}^* = \{x_k^*\}$, import taxes, $\boldsymbol{t}^* = \{t_k^*\}$, and non-revenue trade barriers, $\boldsymbol{\tau}^* = \{\tau_k^*\}$, that maximize national welfare subject to balanced trade:

$$\begin{aligned} \{\boldsymbol{x}^*,\ \boldsymbol{t}^*,\ \boldsymbol{\tau}^*\} \quad &= \quad \arg\max\ V_i(Y_h, \boldsymbol{P}_h) \\ \text{s.t.} \qquad & \sum_{k=1}^{K} X_{fh,k} = \sum_{k=1}^{K} X_{hf,k}. \end{aligned} \tag{1}$$

The above policy problem accounts for various general equilibrium margins that are absent in many standard trade policy frameworks. These margins include the *cross-price elasticity effects* that are typically eliminated with the adoption of Cobb-Douglas preferences, and *wage effects* that are usually precluded by adopting a framework in which tariffs have no impact on factor prices.[12] In the presence of these general equilibrium effects, a tariff imposed in one particular sector can affect the entire vector of sectoral demands and prices.

The optimization problem characterized above is plagued by the curse of dimensionality. It involves countries exchanging many goods whose prices depend on the entire vector of net imports through their effects on wages. Hence, solving for the optimal tax vector involves characterizing the best policy response in sector k as a function of the policy vectors in all other sectors. Even then, there remains the task of verifying whether the best response functions intersect at a unique global optimum. Given their complicated nature, these general equilibrium considerations have been traditionally avoided by the trade policy literature, which restricts attention to partial equilibrium environments or two-good (i.e., one sector) economies.

Our approach to solving Problem 1 involves several steps. Trivially, in the presence of revenue generating taxes, optimal NRTBs will be zero—NRTBs attain importance only when other policy instruments are unavailable (see Section 4.3). Knowing this, we first solve for the vector of optimal import tariffs conditional on zero export tax. The solution to the first step turns out to be a uniform vector of tariffs. Then, we prove the uniformity property holds under an arbitrary vector of export taxes. Finally, knowing that the vector of optimal tariffs is uniform, we solve for the optimal trade policy schedule, which consists of a vector of non-uniform (sector-specific) export taxes/subsidies, uniform import tariffs, and zero NRTBs. In what follows, we describe our approach in full detail.

[12] Wages are unaffected by tariffs if it is assumed that all countries produce a positive amount of a *linear good* at a fixed unit labor requirement. This assumption ensures that tariffs in one sector do not affect the cost of production and prices in other sectors.

Step 1: *Optimal Import Tariffs*

As a first step, we solve a restricted version of Problem 1. That is, we characterize the vector of optimal tariffs for an arbitrary vector of export taxes. Before describing our strategy, one should bear in mind that our analysis covers an environment that features a discrete number of sectors that are either differentiated or homogeneous. Characterizing the optimal tariff across differentiated sectors is a smooth problem. Homogeneous sectors are however subject to knife edge equilibria, and solving for the optimal tariff across these sectors involves a non-smooth welfare maximization problem. One approach to smoothing the problem in such cases is to assume a continuum of (measure zero) sectors.[13] Here we employ an alternative strategy. We characterize the optimal policy vector for a countable number of sectors with an arbitrary vector of θ_k's. Then, to determine the optimal policy for homogeneous sectors, we calculate the solution in the limit where $\theta_k \to \infty$.

Starting with zero export tax, we characterize the vector of optimal tariffs in three basic steps. First, we observe that casting the problem in terms of f.o.b. trade values rather than trade shares (which is the traditional approach) greatly simplifies the problem. Second, for a given sector, we show that any solution to the first order condition (FOC) consists of vectors that include only aggregate variables. The second step, therefore, establishes uniformity. Given the uniform structure, solving for the vector of optimal tariffs becomes rather straightforward. In particular we show that (i) a unique vector of uniform tariffs solves the FOC, and that (ii) this unique vector constitutes a global maximum. Our findings are summarized in the following Lemma.

Lemma 1. *For an arbitrary set of export taxes/subsidies, the optimal import tariffs are unique and uniform across all sectors. Moreover, for zero export taxes/subsidies,*

[13] This approach was initially developed by Dornbusch et al. (1977), and has been employed by Opp (2010) and Costinot et al. (2015) in the context of optimal trade policy.

the optimal import tariff is equal to Home's average export elasticity:

$$t_1^* = \dots = t_K^* = \frac{1}{\epsilon_{hf}}, \tag{2}$$

where $\epsilon_{hf} \equiv \partial \ln \lambda_{hf} / \partial \ln w$ *represents the elasticity of Foreign's aggregate demand for Home's exports.*[14]

This Lemma states that the optimal import tax reflects Home's collective export monopoly power across all sectors. Noting that $\lambda_{hf} = \sum_{k=1}^{K} \alpha_{f,k} \lambda_{hf,k}$ (where $\alpha_{f,k} \equiv Y_{f,k}/Y_f$), Lemma 1 implies the following formula for optimal tariffs:

$$\bar{t}^* = \frac{1}{\tilde{\theta}_{hf} \lambda_{ff} + \sum_{k=1}^{K} \left(\frac{X_{hf,k}}{X_{hf}} - \alpha_{f,k} \right) \frac{\partial \ln \alpha_{f,k}}{\partial \ln w}}, \tag{3}$$

where $\tilde{\theta}_{hf} \equiv \sum \alpha_{f,k} \frac{\lambda_{fh,k}}{\lambda_{fh}} \frac{\lambda_{ff,k}}{\lambda_{ff}} \theta_k$ could be viewed as the weighted average elasticity of Foreign demand for Home's exports. The above formula reduces our original $K+1$-dimensional welfare optimization problem to a simple system of 2-equations$\times$2-unknowns that is solvable with a basic matrix inversion. In that regards, our formula greatly simplifies the computational analysis of trade policy—a direction which we will explore in Section 5. Next, however, we use the above formula to highlight the determinants of the optimal import tax.

The first term in the denominator Equation 3, $\tilde{\theta}_{hf} \lambda_{ff}$, corresponds to *wage* effects (i.e., $\partial w_f / \partial t_k$), which reflects the ability of import policy to manipulate the vector of Foreign prices through the economy-wide wage. Previously, Gros (1987) and Alvarez and Lucas (2007) have characterized the role of wage effects, but only in the context of a single sector economy. Notably, in the presence of differentiated sectors, these effects retain quantitative importance even when Home is a small open economy (i.e., $\lambda_{ff} \approx 1$).

The second term in Equation 3, $\sum_{k=1}^{K} \left(\frac{X_{hf,k}}{X_{hf}} - \alpha_{f,k} \right) \frac{\partial \ln \alpha_{f,k}}{\partial \ln w}$, corresponds to "cross-price elasticity" effects. This is perhaps the most novel feature of our optimal tariff formula, brining to light classic arguments dating back to

[14] Proof is provided in Appendix A.

Graaff (1949). In general, under homothetic preferences, cross-price elasticity effects lower the export market power of the Home country, leading to a lower optimal tariff. Using the example of CES preferences, we elaborate on these effects in Appendix C. Furthermore, while cross-price elasticity effects are often neglected in computational analyses, we highlight their empirical significance in Section 5.2.

The role of cross-price elasticity effects are more nuanced under non-homothetic preferences. For example, suppose income effects are large enough such that $\frac{\partial \ln \alpha_{f,k}}{\partial \ln w} > 0$ for income-elastic sectors.[15] If Home has a comparative advantage in income-elastic sectors (i.e., $\frac{\partial \ln \alpha_{f,k}}{\partial \ln w} > 0$ iff $\frac{X_{hf,k}}{X_{hf}} > \alpha_{f,k}$), the cross-price elasticity term in the optimal tariff formula will be positive, thus reducing the optimal tariff level. The intuition is straightforward: tariffs lower Foreign's income and shift expenditure away from income-elastic goods. If Home exports predominantly in income-elastic sectors, it would set lower tariffs to counter these consumption-shifting effects.

The uniformity result in Lemma (1) extends earlier findings in Costinot et al. (2015) and Opp (2010) to a setting where trade elasticities are non-uniform and demand is non-separable across sectors.[16] The uniformity result by itself is perhaps less interesting as it hinges on restriction R3. In particular, while it holds under any general demand structure or with any number of countries, it disappears once we relax R3. However, the uniformity result points to an interesting corollary. It suggests that accounting for traditionally overlooked general equilibrium interactions will always dampen the optimal variation in sectoral tariffs.

To provide intuition about the uniformity result, we can appeal to the non-substitution theorem (Koopmans 1951). Restrictions R1-R3 satisfy the assumptions necessary for this theorem, which states that the relative prices of a country's outputs *net of taxes* are uniquely determined by the relative

[15] Note that $\frac{\partial \ln \alpha_{f,k}}{\partial \ln w} = \frac{\partial \ln \alpha_{f,k}}{\partial \ln p_{ff,k}} \frac{\partial \ln p_{ff,k}}{\partial \ln w} + \frac{\partial \ln \alpha_{f,k}}{\partial Y_f} \frac{\partial \ln Y_f}{\partial \ln w}$, where as noted earlier w denotes the Foreign wage and $p_{ff,k}$ is the local price of Foreign varieties in sector k.

[16] Additionally, compared to Costinot et al. (2015), our optimal tariff formula eliminates the Lagrange multipliers, making it more practical for computational purposes.

sectoral efficiencies. Therefore, the relative price of the imported goods on the world market are independent of the import tariff structure. On the other hand, uniform tariffs retain the parity between domestic and world relative prices of imports, thereby eliminating one of the distortions created by tariffs. To be exact, note that the marginal effect of a sector-wide tariff, t_k, on welfare is exactly proportional to its marginal effect on relative wage:[17]

$$\frac{d \ln V_h}{dt_k} = \Gamma(t_k, \mathbf{X}) \frac{d \ln w}{dt_k},$$

where $\mathbf{X}$ is a vector of aggregate variables, with t_k being the only term in $\Gamma(t_k, \mathbf{X})$ that depends on k. Hence, given that $\Gamma(t, \mathbf{X}) = 0$ has a unique solution, uniformity is a necessary condition for optimality.

Step 2: *The Full Optimal Policy Schedule*

We now turn to characterizing the full vector of optimal trade taxes. Lemma 1 supplemented with the Lerner symmetry theorem paves the way to deriving the full optimal policy schedule. Given that optimal import taxes are uniform and non-revue barriers are zero for an arbitrary vector of export policies (Lemma 1), the optimization problem (1) reduces to finding a vector of export taxes $\boldsymbol{x}^*$ given an arbitrary uniform tariff t^*:

$$\begin{aligned} \{\boldsymbol{x}^* \mid \boldsymbol{t} = t^*,\ \boldsymbol{\tau} = 0\} \quad &= \quad \arg\max\ V_i(Y_h, P_{h,1}, ..., P_{h,K}) \\ \text{s.t.} \qquad & \sum_{k=1}^{K} X_{fh,k} = \sum_{k=1}^{K} X_{hf,k}. \end{aligned} \tag{4}$$

Due to Lerner's Symmetry, the optimal policy is indeterminate, but *unique* up to a uniform tariff, t^*.[18] Specifically,

Proposition 1. *The optimal trade tax/subsidy schedule consists of zero non-revenue barriers, a uniform tariff, and variable (sector-specific) export*

[17] For an exact description of $\Gamma(.)$, see the proof of Lemma 1 in Appendix A.

[18] Figure 1 illustrates the indeterminacy of the optimal tax for a one sector economy under various values of θ.

taxes/subsidies. For the case where $\alpha_{i,k}$'s are constant (i.e., assuming a Cobb-Douglas aggregator), the optimal policy schedule is uniquely characterized by the following formula up to a uniform tariff t^*:

$$\begin{cases} \tau_k = 1 \\ t_k = t^* \\ x_k = \frac{1+1/\theta_k \lambda_{ff,k}}{1+t^*} - 1 \end{cases} \qquad \forall k,$$

where $\theta_k \lambda_{ff,k} = \frac{\partial \ln \lambda_{hf.k}}{\partial \ln(1+x_k)}$ represents the elasticity of Foreign's demand for Homes's exports in sector k.[19]

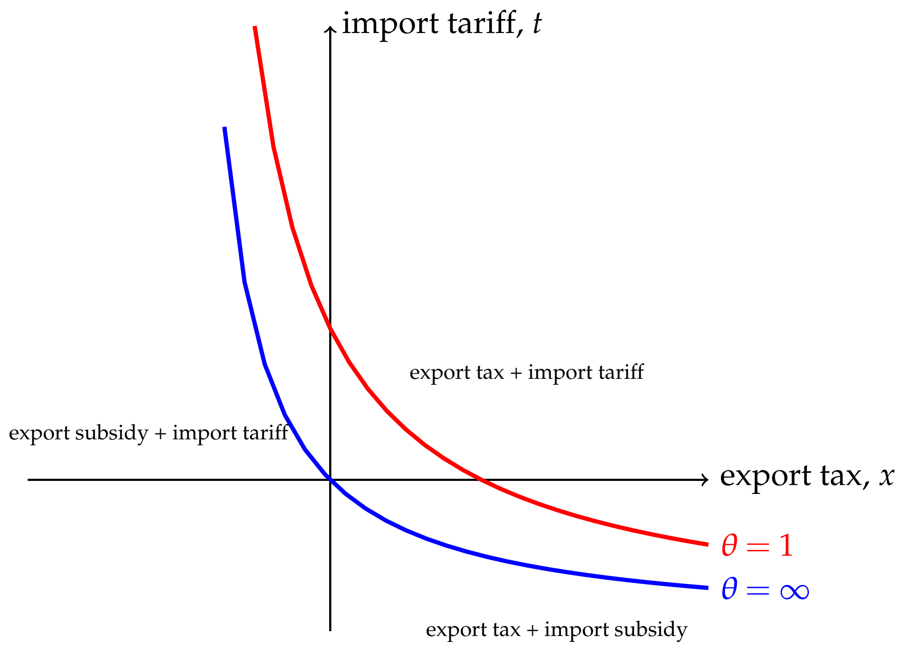
The *unconstrained* optimal policy structure reflects the potency of each policy instrument in manipulating the terms of trade. The optimal export tax, which is the most potent instrument varies across sectors with Home's sector-level monopoly power on the world markets. The optimal import tax is less potent, and set uniformly across sectors. Non-revenue trade barriers are the least potent, and set to zero when other policy instruments are available.

Recall that we analytically characterized the optimal policy schedule, because the most general formulations available, Costinot et al. (2015), only applies to a special case where the trade elasticity, θ_k, is uniform across sectors.[20] As one can see from Proposition 1, non-uniformities in the trade elasticity are indeed the main driver of sectoral variation in optimal policy. Moreover, as shown in Appendix C.1, the formula described by Proposition 1 nests the limit-pricing scheme in Costinot et al. (2015) as a special case where $\theta_k \to \infty$. Therefore, by itself, Proposition 1 presents a major advance toward better understanding the structure of optimal policy in multi-sector gravity models. Also for computational purposes, our formula reduces the optimal tax problem from a high-dimensional optimization to a simple ma-

[19] Proof is provided in Appendix B.

[20] Costinot et al. (2015) characterize the conditional variation in the optimal trade taxes for a given vector of wage and Lagrange multipliers. Their results for the case of finite θ's are, however, derived under a CES utility aggregator.

Figure 1: The optimal trade tax schedules for various values of θ.



Note: The figure is plotted based on the assumption that Home is small compared to the Foreign (i.e. rest of the world): $\lambda_{ff} \approx 1$.

trix inversion problem.

4 Interdependence of Policies

When sectors are interrelated through various general equilibrium margins, trade policies become interdependent. In particular, the optimal tax in one sector depends on the taxes and subsidies applied in other sectors. In this section we analyze the interdependence of tariffs across sectors, import and export taxes/subsidies, and tariffs and non-tariff measures.

4.1 Interdependence of import tariffs across sectors

First, we characterize a systematic pattern of interdependence between sectoral import tariffs. To this end, we analyze the effect of a tariff liberalization in a subset of sectors (namely, R) on optimal tariffs in the remaining sectors with no restriction on their trade policy space–i.e., unbound sectors. This exercise is reminiscent of the pattern of trade liberalization under the GATT and the WTO that features substantial tariff cuts in some sectors and unbound or loosely-bound tariffs in other sectors.[21]

We find that import tariffs are generally complementary across sectors. More specifically,

Proposition 2. [Tariff Complementarity] *Liberalizing tariffs in a subset of sectors lowers the optimal tariff in the other (unbound) sectors. Moreover, the optimal tariffs in the unbound sectors are uniform and is given by*

$$t_R^* = \frac{1}{\epsilon_{hf} + \sum_{g \in R} \frac{X_{fh,g}}{X_{fh}} \left[1 + \omega_g\right]} < \frac{1}{\epsilon_{hf}},$$

where $\omega_g \equiv -\partial \ln X_{fh,g} / \partial \ln w > 0$.[22]

[21] Beshkar et al. (2015) show that this is specially true for developing country members of the GATT/WTO that have committed to tariff cuts in a subset of sectors while retaining substantial policy space in other sectors.

[22] Proof is provided in Appendix D.

To gain intuition, note that two forces contribute to these complementarity effects. First, lowering tariffs in unbound sectors is attractive because it decreases the tariff-induced price distortions. Second, partial liberalization decreases national income (through tariff revenue cuts) and increases the Foreign wage. Both adjustments decrease the volume of trade in unbound sectors and, thus, the marginal benefits from tariffs.

The finding that tariff liberalization in a subset of sectors encourages lower tariffs in unbound sectors is a novel result that has an important implication about the design of trade agreements. In Section 5, we examine the quantitative significance of these policy interdependencies.

Going one step further, we can characterize the optimal import tariff in sector k as a function of the vector of applied tariffs in other sectors. In particular, the FOC corresponding to sector k's tariff, implies the following best response formula (see Appendix E for derivation):

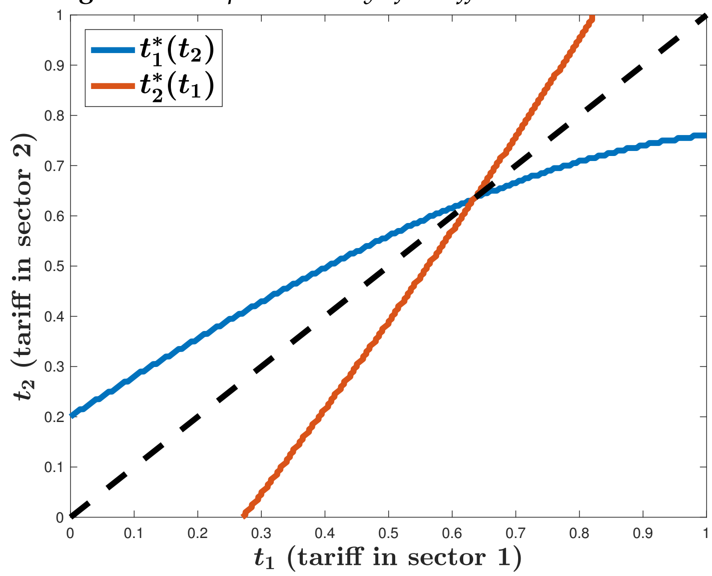
$$t_k^*(t_1,...,t_K) = \frac{1 + \sum_{g \neq k} t_g \frac{X_{fh,g}}{X_{fh}} \left[1 + \omega_g\right]}{\epsilon_{hf} + \sum_{g \neq k} \frac{X_{fh,g}}{X_{fh}} \left[1 + \omega_g\right]}.$$

The above formula, also points to the complementarity of import tariffs: The optimal tariff in sector k increases with the weighted average tariff imposed on the remaining $K-1$ sectors. Figure 2 illustrates these arguments in a two-sector economy.

4.2 Interdependence of import and export policies

Our second finding sheds new light on the interdependence between export and import policies. In the view of the Lerner's Symmetry, it is widely believed that import and export taxes are perfect substitutes. Propositions 1, however, challenge this belief. Proposition 1 states that while optimal tariff is uniform across sectors, the optimal export tax is non-uniform. Hence, applying the Lerner's Symmetry, the welfare effects of the optimal tariff can be exactly replicated with a uniform export tax, whereas the outcome

Figure 2: *Complementarity of Tariffs Across Sectors*



Note: The best-response tariff schedule ($t_1^*(t_2)$ and $t_2^*(t_1)$) are depicted for the Home country in a two-sector framework. The cross-sector utility aggregator is Cobb-Douglas and the economy is simulated using the following parameter values: $\theta_1 = 6$; $\theta_2 = 2$; $\alpha_{i,1} = \alpha_{i,2} = 0.5$; $L_h = L_f = 1$; $\tau_{hf,k} = \tau_{fh,k} = 1.5$; and $A_{j,k} = 1$.

attained under the optimal export policy is unattainable with any import policy alternative. That is to say, import policy is at most a weak substitute for export policy—a result highlighted in the following proposition.

Proposition 3. [Imperfect Substitutability of Import and Export Policies] *Import tariffs are an imperfect substitute for export taxes, i.e. governments could achieve higher levels of welfare through export policy than import policy alone.*

Note that the above result applies to a wide range of canonical long-run competitive trade models. In the long-run models, import tariffs can only utilize the economy-wide market power by reducing demand for foreign factors of production, thereby depressing the foreign wage and import prices. Export policy, however, affects the terms of trade through two distinct channels. First, in a similar way to import tariffs, export policy has an economy-wide impact on wages. Moreover, *for a given wage,* export policy can change foreign prices since foreign country's import demand is less than perfectly elastic. Therefore, export policy is more *potent* than import policy as an instrument for manipulating terms of trade.

Finally, Proposition 3 sheds new light on the WTO's strong restrictions on export subsidies. This restriction is puzzling from the perspective of the classical partial equilibrium theories, which purport that export subsidies deteriorate the imposing country's terms of trade and, thus, negotiating such a ban could not serve an economic objective. Moreover, a ban on export subsidy is hard to reconcile with the view that import and export policies are perfect substitutes. Proposition 3, however, implies that restricting export policy alone-–i.e., without any restriction on import policy—can increase the joint welfare of the trading partners.

4.3 Interdependence of NRTBs and Trade Taxes

Suppose that governments are unable to use tariffs due to their obligations under international trade agreements. Are there any unilateral welfare gains for an importing country from erecting other (potentially concealed) trade barriers that restrict imports without generating any revenues for the government? Under standard *partial-equilibrium*—as well as *one-sector general-equilibrium*—models of trade policy, the answer to this question is negative. We, however, show that in a multi-sector general equilibrium framework, NRTBs are in fact beggar-thy-neighbor policies that could improve the importing country's welfare in expense of the exporting countries.

In response to tariff cuts, many countries have opted for non-tariff barriers, most of which do not generate any revenues for the governments. As emphasized by Goldberg and Pavcnik (2016), a true measure of trade policy must take into account the prevalence of non-tariff measures, especially after the implementation of negotiated tariff cuts. Therefore, to analyze the impact of trade policy, it is imperative to understand the incentives of the governments in choosing NRTBs, the likely pattern of NRTBs chosen by the governments, and their welfare implications.

The optimal NRTBs are chosen to solve the following problem:

$$\{\boldsymbol{\tau}^* \mid \boldsymbol{t} = \boldsymbol{x} = 0\} = \arg\max \; V_i(Y_h, \boldsymbol{P}_h),$$
$$\text{s.t.} \sum_{k=1}^{K} X_{fh,k} = \sum_{k=1}^{K} X_{hf,k}.$$

In Appendix F, we formally establish that, unlike revenue-generating tariffs, optimal NRTBs cannot be both non-zero and uniform. In fact, in stark contrast to partial equilibrium models, NRTBs will be prohibitively large in high-elasticity (i.e., relatively homogeneous) sectors, and zero in others. More specifically,

$$\tau_k^* = \begin{cases} 1 & \text{if } \epsilon_{fh,k} \le \epsilon_{hf} + \epsilon_{fh}, \\ \infty & \text{if } \epsilon_{fh,k} > \epsilon_{hf} + \epsilon_{fh}, \end{cases}$$

where $\epsilon_{ji} \equiv |\frac{\partial \ln \lambda_{ji}}{\partial \ln w}|$ and $\epsilon_{fh,k} \equiv |\frac{\partial \ln \lambda_{fh}}{\partial \ln w}| = \theta_k \alpha_{h,k} \lambda_{hh,k}$. The above characterization indicates that in a one-sector model the optimal NRTB is always zero—i.e., $\epsilon_{fh,k} = \epsilon_{fh} < \epsilon_{fh} + \epsilon_{hf}$. However, in an environment with heterogeneous sectors, the optimal NRTB will be positive if sector k is sufficiently large (high-$\alpha_{h,k}$), sufficiently productive (high-$\lambda_{hh,k}$), and features a sufficiently large trade elasticity, θ_k.[23] We summarize these arguments in the following proposition.

Proposition 4. *Absent revenue generating taxes, it is optimal to impose a prohibitively large NRTB on sectors with sufficiently high trade elasticities.*[24]

In Section 5, we calibrate our model to actual sectoral trade data and find that four sectors in the US economy display such properties: Wheat, Rice, Diary, and Apparel. It is, therefore, optimal for the US to set prohibitively

[23] In theory, one can easily construct an example where sector k is subject to a positive non-revenue tariff. One example corresponds to a K-sector economy where α's are uniform, and Home and Foreign are symmetric. Hence, $X_{ji,k} = X_{ji}/K$ for all k, and for the non-revenue tariff to be positive we should have $\theta_k > 2\sum_{k'=1}^{K} \frac{\theta_{k'}}{K}$. That is, if sector k has a trade elasticity double the simple average elasticity, the Home government will impose a positive tariff on that sector, even when tariff revenue bears no value to the government.

[24] Proof is provided in Appendix F.

high import barriers on Wheat, even without revenue considerations. The logic behind this result can be stated as follows. Restricting imports in high-elasticity (and productive) sectors lowers the foreign wage rate and the import price in all other sectors. In practice, these wage effects alone can be large enough to offset the welfare loss due to price increase in the trade-restricted sector.

By showing that *revenue generation* is not a necessary condition for optimality of protectionist policies, this result counters the skepticism about the relevance of the terms of trade theories for policymaking on the ground that governments in advanced countries do not value tariff revenues as much as consumer or producer surplus. In fact, NRTBs correspond to an extreme case where trade barriers generate zero revenue. Alternatively, one may assume that governments assign a political weight, η, (potentially different from one) to tax revenues by maximizing $V(w_h L_h + \eta R_{fh}, \mathbf{P})$. In that case, it is straightforward to show that optimal tariffs are non-uniform. These arguments could shed light on the difference between the trade policy adopted by countries at different development stages that assign different values to tariff revenues.

5 Quantitative Analysis

So far we have provided an analytical characterization of optimal trade policy under general equilibrium interactions. In the interest of simplicity, the previous analyses of optimal trade policy have assumed away these interactions. One, therefore, wonders how important these general equilibrium considerations are in practice. In this section we use sector-level data on trade, production, and consumption values to address this question. Overall, we find that general equilibrium considerations are quantitatively significant.

We calibrate our model using sector-level trade elasticities and applied tariffs to match US and Chinese trade shares with the rest of the world across 33 sectors. The data is sourced from the Global Trade Analysis Project database (GTAP 8) which provides sector-level trade, production, and tariff data. Ossa (2014) eliminates trade imbalances from this data set, and estimates a sector-specific trade elasticity for each of the 33 sectors in the sample. Focusing on this data allows us to contrast the predictions of our supposedly long-run model to that of Ossa (2014), which features short-run profit shifting effects.

First, we consider Home to be the US and Foreign to be an aggregate of the rest of the world (ROW). For identification purposes, we impose the following parametric restriction on preferences: $U_i(\mathbf{Q}) = \Pi_{k=1}^{32} Q_{i,k}^{\alpha_{i,k}}$—later, we relax this restriction to highlight the role of cross-price elasticity effects. Given the data on population size, L_i, and sectoral expenditure shares, $\alpha_{i,k}$, the parameters necessary to compute the optimal tariffs are the sector-level trade shifters, $A_{ji,k}$ and sector-level trade elasticities, θ_k. The sector-level trade elasticities are borrowed from Ossa (2014). We assume that $A_{ji,k} = A_{j,k}\tilde{\tau}_{ji}$, where $A_{j,k}$ denotes the productivity component of the trade shifter, and $\tilde{\tau}_{ji}$ denotes the transport cost component, which is symmetric with $\tilde{\tau}_{ii} = 1$. We pin down sector-level productivities, $A_{j,k}$, by matching sector-level production shares, and set the transport cost parameter, $\tilde{\tau}_{ji}$, to match trade shares. Given the calibrated parameters and the sector-level trade elasticities, we compute the optimal US trade tax schedule using Lemma 1 and Proposition 1. To select a unique tax schedule from the set of optimal solutions, we invoke a clause in the US constitution that prohibits export taxes, and assume zero export subsidy on *Animal Products*, which entails a positive optimal subsidy on all other sectors.[25]

Without Propositions 1, computing the optimal policy vector involves the method of mathematical programming with equilibrium constraints (MPEC) developed by Su and Judd (2012) (see Ossa 2014 for an application). Compared to the standard MPEC method, our analytical characterization of the optimal trade tax reduces computation time considerably—applying

[25]The Export Clause of the U.S. Constitution (Article I, Section 9, Clause 5), states that *"No Tax or Duty shall be laid on Articles exported from any State."* For a legal analysis of this clause see Lunder (2012).

Proposition 1 we can calculate the optimal tax vector 27-times faster. Despite these differences in the rate of convergence, both approaches deliver the exact same optimal trade tax schedule.

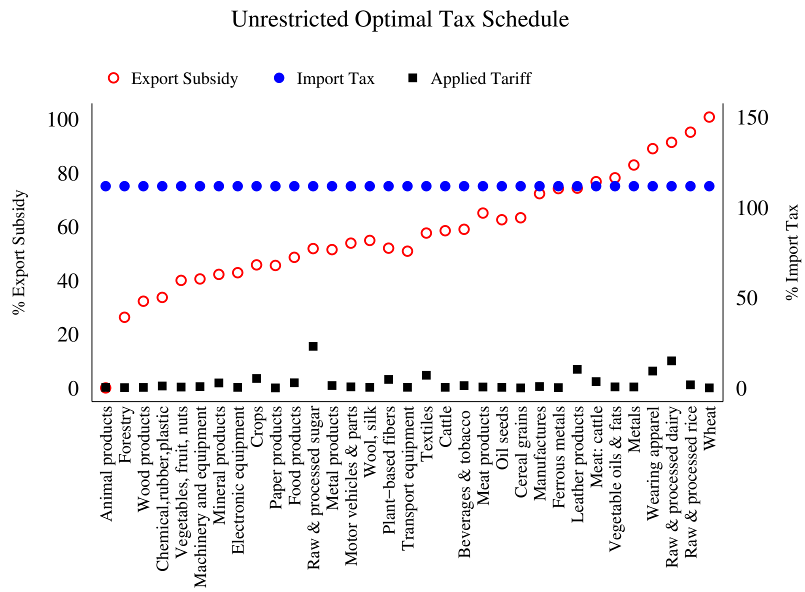
The top panel in Figure 3 displays the unrestricted optimal trade tax schedule for the US, and compares it to applied tariffs across the 33 sectors in the sample. Sectors are sorted according to their trade elasticity θ_k, with low-θ (more differentiated) sectors to the left and high-θ (more homogeneous) sectors to the right. The optimal tax schedule for the US consists of a uniform 111.7% tariff and a varying export subsidy averaging at 57.9%. The optimal export subsidy varies with (i) the trade elasticity, θ_k, and (ii) the comparative advantage of the US, $A_{h,k}/A_{f,k}$. A simple variance decomposition implies that variations in θ_k account for 99.4% of the variation in the optimal US export subsidy. This outcome simply reflects that, factually, traded sectors are far from homogeneous—as implied by our theory the role of comparative advantage diminishes the more differentiated the traded sectors.

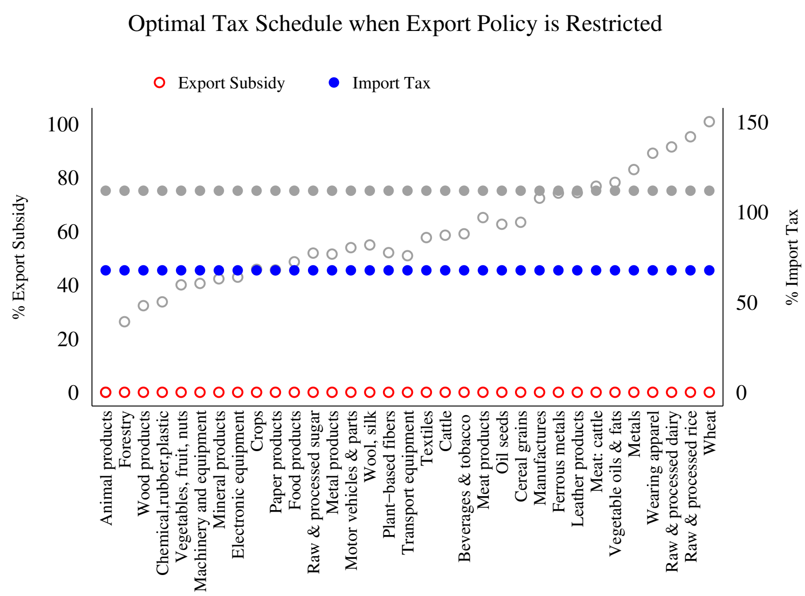
5.1 Interdependence of Policies: The Effect of Incomplete Agreements

In light of our theoretical results, we now study the effect of incomplete agreements that restrict only a subset of policies and leaves the rest to the discretion of the governments. To this end, we analyze how restricting a subset of policies affects the optimal level of unrestricted policies by conducting two counter-factual policy scenarios: (1) a negotiated ban on export subsidies and no restriction on tariffs; (2) a negotiated ban on export subsidies and tariff liberalization in a subset of sectors.

If export subsidies were prohibited, the optimal import tariff would be described by Lemma 1. The bottom panel of Figure 3 illustrates the optimal tax schedule for the US when export subsidies are banned and set to zero. In that case, the optimal policy schedule consists of a uniform 67.6% import tariff, which is 44% below the unrestricted uniform tariff. Considering our

Figure 3: *U.S. optimal trade policy*





Note: Sectors are sorted based on their trade elasticity θ_k – the highest-θ sectors are to the right. All optimal tax rates are computed given factual applied tariffs in the rest of the world. The uniform noncooperative U.S. tariff is 66%.

theory, this is not a trivial reduction in import tariffs. In fact, a ban on export subsidies has real effects, and reduces the overall level of protection (we will compute these welfare effects in the following section). By contrast, in a model with only one import and one export good, a ban on export subsidy would have no impact on the overall level of protection.

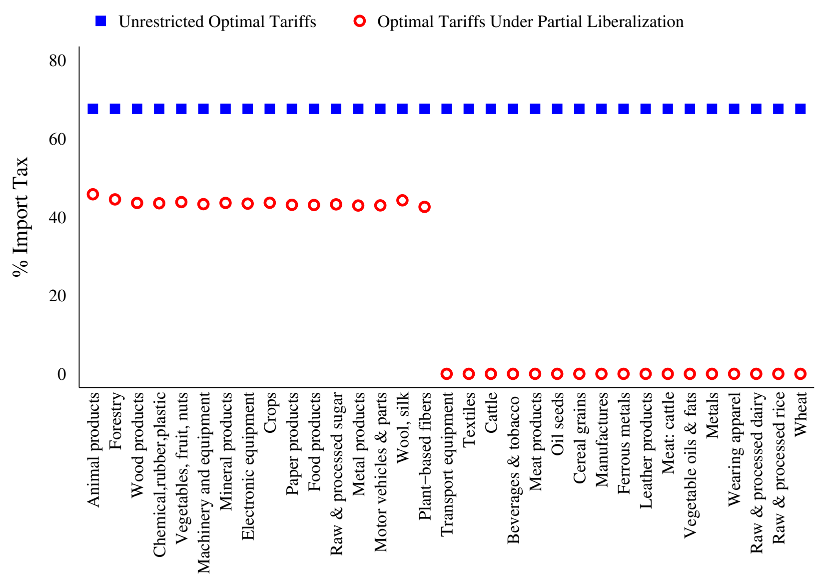
In the second scenario, in addition to a ban on export subsidies, the governments negotiate zero tariffs in half of the sectors with the highest elasticity. Under this scenario, we compute the vector of optimal US import tariffs in the remaining unbound sectors. As shown by Proposition 2, due to complementarity between sectoral tariffs, negotiated liberalization in a subset of sectors leads to voluntary (i.e., unilateral) tariff cuts in unbound sectors. These complementarity effects are displayed in Figure 4, where the optimal tariff on unbound (low-θ_k) sectors drops to about 43%, down from 67.5%. Finally, if all revenue generating instruments are restricted, the optimal US policy will involve prohibitively high NRTBs on the Wheat, Rice, Diary, and Apparel sectors.

To put these numbers in perspective, notice that the optimal tariff levels we compute are close in magnitude to those measured by Ossa (2014) and Broda et al. (2008). The significant magnitude of the computed optimal tariffs in our framework indicates that the general equilibrium effects, which are assumed away in previous calculations of optimal tariffs, are also an important determinant of optimal policy.

5.2 The Role of Cross-Price Elasticity Effects

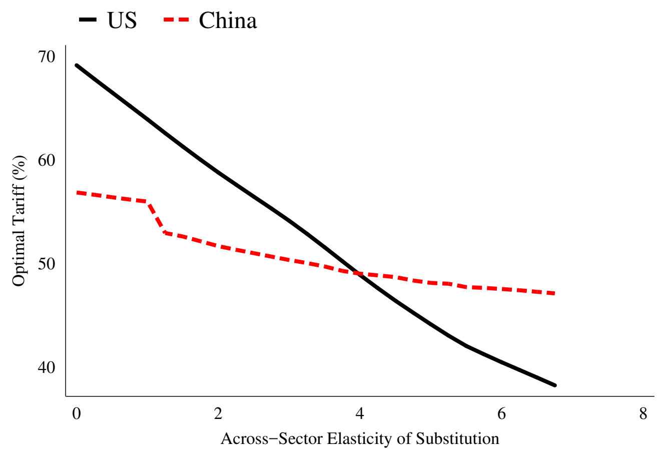
Our previous calibration exercise assumed away cross-elasticities by adopting a Cobb-Douglass utility aggregator across sectors. This assumption is quite standard, with many studies adopting it when performing trade policy analysis (e.g., Caliendo and Parro 2014; Ossa 2014). However, as noted in Subsection C, accounting for cross-price elasticity effects may lower Home's aggregate monopoly power, leading to lower optimal tariffs. Here, we try to determine the quantitative importance of these (often ignored) general

Figure 4: *Complementarity of import tariffs across sectors.*



Note: Sectors are sorted based on their trade elasticity θ_k – the highest-θ sectors are to the right. We restrict import tariffs to be zero in 16 sectors with the highest θ_k and compute the optimal import tariff in the renaming 17 sectors.

Figure 5: *Optimal tariff vs. cross-price elasticity effects*



equilibrium effects. To this end, we relax the Cobb-Douglass assumption, and calibrate our model under a CES utility aggregator across sectors, i.e., $U_i\left(\mathbf{Q}\right) = \left[\sum_k \alpha_{i,k} Q_{i,k}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$. We perform the calibration for various values of σ, computing the optimal tariff corresponding to each case. Figure 5 plots the implied optimal tariff levels for different values of σ, for both the US and China.

As predicted by our theory, optimal tariffs are lower the stronger the underlying cross-price elasticity effects. More importantly, these variations are non-trivial—activating the cross-price elasticity effects, lowers the optimal US tariff from around 70% to below 40%. The US optimal tariff level is higher than China under a zero cross-sector elasticity ($\sigma = 0$), but lower under a sufficiently high σ. The intuition is similar to what we provided earlier in Subsection C. Namely, that US optimal tariffs are more sensitive to the cross-price elasticity, σ, because the US economy displays a higher degree of sectoral specialization.

5.3 Welfare Impact

Thus far, we have shown that the optimal trade taxes resulting from general equilibrium considerations are sizable. Here, we demonstrate that the welfare effects of these general equilibrium taxes are also non-trivial. To this end, we use our calibrated model to conduct multiple counter-factual welfare analyses. First, we compute the welfare effects of a multilateral tariff liberalization between the US and ROW. To this end, we compare aggregate welfare in the factual equilibrium to that in a counter-factual equilibrium with zero applied tariffs. The results (reported in Table 1) indicate that tariff liberalization raises US welfare by only 0.68%, and has a rather insignificant effect on the ROW.[26] These numbers could be used as benchmarks to evaluate the effect of other policies. The second policy we consider involves a unilateral policy reform in which the US uniformizes sectoral tariffs—i.e., all sector are subjected to a trade-weighted average applied tariff of 1.4%. This reform will increase US welfare by 0.86%, while making the ROW worse off by 0.13%.

In a second set of counterfactual exercises, we compute the welfare effects of optimal unilateral policies. First, we compute the welfare effects that result from the imposition of unilaterally optimal US tariffs. Second, we compute the welfare effects that result from the imposition of unilaterally optimal US trade taxes, which involve both import tariffs and export subsidies. Our findings, which are reported in the last two rows of Table 1, can be summarized as follows. A unilaterally optimal *import* policy could raise the real per capita income of the US by 3.48%. In comparison, a unilaterally optimal trade policy that involves export subsidies could raise the real per capita income of the US by 3.55%. However, the application of optimal export subsidies –while more beneficial to the US– would impose a greater negative externality on the ROW. As seen in Table 2, similar effects arise when China imposes unilaterally optimal trade taxes on the ROW.

[26]The ROW experiences a negligible loss from multilateral liberalization because it applies tariffs to the US that are 4.5-times larger than those applied by the US.

Table 1: *The Effect of U.S. trade Policy on National and Global Welfare*

	% Change in Welfare Relative to Factual	
Counterfactual Scenario	U.S.	R.O.W.
Multilateral Tariff Liberalization	0.68%	-0.01%
Uniformizing Applied Tariffs	0.86%	-0.13%
Optimal Unilateral Import Policy	3.48%	-1.18%
Optimal Unilateral Trade Policy	3.55%	-1.26%

Note: This table reports changes in welfare when moving to four counterfactual scenarios. *Multilateral tariff liberalization* is when both the US and Rest of World (ROW) eliminate their applied tariffs. *Uniformizing applied tariffs* corresponds to the case where the US unilaterally revises its trade policy by applying a uniform 1.4% tariff across all sectors (which is equal to its trade-weighted average applied tariff). *Optimal unilateral import policy* corresponds to the case where the US imposes a unilaterally optimal tariff. *Optimal unilateral import policy* corresponds to the case where the US imposes an optimal trade tax that includes a uniform tariff a sector-specific export subsidies.

Table 2: *The Effect of China's trade Policy on National and Global Welfare*

	% Change in Welfare Relative to Factual	
Counterfactual Scenario	China	R.O.W.
Multilateral Tariff Liberalization	0.19%	0.11%
Uniformizing Applied Tariffs	1.05%	-0.13%
Optimal Unilateral Import Policy	2.26%	-0.86%
Optimal Unilateral Trade Policy	2.46%	-0.98%

Note: This table reports changes in welfare when moving to counterfactual scenario. *Multilateral tariff liberalization* is when both China and the Rest of World (ROW) eliminate their applied tariffs. *Uniformizing applied tariffs* corresponds to the case where China unilaterally revises its trade policy by applying a uniform 10.7% tariff across all sectors (which is equal to its trade-weighted average applied tariff). *Optimal unilateral import policy* corresponds to the case where China imposes a unilaterally optimal tariff. *Optimal unilateral import policy* corresponds to the case where China imposes an optimal trade tax that includes a uniform tariff a sector-specific export subsidies.

6 Concluding Remarks

We revisit the problem of optimal trade taxation in a general equilibrium model that features many differentiated sectors. Our analysis covers a large class of canonical trade models—including Costinot et al.'s (2011) multi-sector version of the Eaton-Kortum model, Chaney's (2008) version of the Melitz model,[27] and Dornbusch et al.'s (1977) Ricardian trade model.

Our analysis reveals two novel trade policy interdependencies: i) Import tariffs are complementary across sectors and ii) Import policy is an imperfect substitute for export policy. The result that import policy is only an imperfect substitute for export policy has an important and novel implication about the design of trade agreements. At a broad level, this result implies that no trade liberalization may be achieved by restricting import policy alone. But importantly, an incomplete agreement that restricts only export policy and leaves import policy unconstrained will result in trade liberalization.

We find that under certain conditions the optimal trade tax schedule involves a uniform import tariff rate and a sector-specific export subsidy that increases with the sector-wide trade elasticity. Our findings shed light on the *cause of the optimality of uniform import tariffs*: Under these perfectly competitive models, a necessary and sufficient condition for optimality of uniform tariffs is for sectoral efficiencies to be invariant to trade policy. As is known from the literature on non-substitution theorem (Koopmans 1951), under these conditions the relative prices of a country's outputs *net of taxes* are uniquely determined by the relative sectoral efficiencies. Therefore, the relative price of the imported goods on the world market are independent of the import tariff structure. On the other hand, uniform tariffs retain the parity between domestic and world relative prices of imports, thereby eliminating one of the distortions created by tariffs. In monopolistically competitive models, in addition to the invariance of sectoral efficiencies to policy, the optimality of uniform tariffs requires conditions that preclude profit shifting

[27] See Appendix G.

and firm-delocation effects.

A surprising result of our analysis is that in general equilibrium with multiple sectors, *revenue generation* is not a necessary condition for optimality of protectionist policies. This result counters the skepticism about the relevance of the terms of trade theories for policy making on the ground that governments in advanced countries do not value tariff revenues as much as consumer or producer surplus. Moreover, this result points to the possibility that negotiated tariff cuts may be partially nullified by the introduction of non-tariff barriers even in the absence of lobbying by the domestic import-competing industries.

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A Proof of Propositions 1 [Optimal Import Tariffs]

We first characterize the optimal import tariff under zero export tax and show that it is uniform across sectors. We then show that the uniformity result continues to hold under an arbitrary set of export taxes/subsidies.

Optimal import tariff given zero export tax/subsidy. First, we write down the first order conditions (FOC), Then, we show that a unique vector of uniform tariffs satisfies the FOC. The FOC corresponding to sector k can be written as:

$$\frac{dV_h}{dt_k} = \frac{\partial V_h}{\partial Y_h}\frac{\partial Y_h}{\partial t_k} + \frac{\partial V_h}{\partial P_{h,k}}\frac{\partial P_{h,k}}{\partial t_k} + \left[\frac{\partial V_h}{\partial Y_h}\frac{\partial Y_h}{\partial \ln w} + \sum_g \frac{\partial V_h}{\partial P_{h,g}}\frac{\partial P_{h,g}}{\partial \ln w}\right]\frac{d\ln w}{dt_k}.$$

Given the balanced trade condition (R2: $X_{fh} = X_{hf}$) the wage effect of a tariff in sector k is given by

$$\frac{d\ln w}{dt_k} = \frac{\frac{\partial \ln X_{fh}}{\partial t_k}}{\frac{\partial \ln X_{hf}}{\partial \ln w} - \frac{\partial \ln X_{fh}}{\partial \ln w}}.$$

Additionally, note that $\frac{\partial \ln X_{hf}}{\partial \ln w} = 1 + \epsilon_{hf}$, where $\epsilon_{hf} \equiv \frac{\partial \ln \lambda_{hf}}{\partial \ln w}$. Roy's identity implies that $\frac{\partial V_h}{\partial P_{h,k}} = -\frac{Y_{h,k}}{P_{h,k}}\frac{\partial V_h}{\partial Y_h}$. Furthermore, $\frac{\partial \ln P_{h,k}}{\partial t_k} = \frac{\lambda_{fh,k}}{1+t_k}$ and $\frac{\partial \ln P_{h,k}}{\partial \ln w} = \lambda_{fh,k}$. Accounting for these relations, the FOC will become

$$\frac{\partial V_h}{\partial Y_h}\left(X_{fh,k} + \tilde{t}_k X_{hf}\frac{\partial \ln X_{fh}}{\partial t_k}\right) - \underbrace{\frac{Y_{h,k}}{P_{h,k}}\frac{\partial V_h}{\partial Y_h}P_{h,k}\frac{\lambda_{fh,k}}{1+t_k}}_{X_{fh,k}\frac{\partial V_h}{\partial Y_h}} + \left[\frac{\partial V_h}{\partial Y_h}\frac{\partial Y_h}{\partial w} + \sum_g \frac{\partial V_h}{\partial P_{h,g}}\frac{\partial P_{h,g}}{\partial w}\right]\frac{dw}{dt_k} = 0,$$

where

$$\tilde{t}_k \equiv \frac{\sum_g t_g \frac{X_{fh,g}}{X_{fh}}\frac{\partial \ln X_{fh,g}}{\partial t_k}}{\sum_g \frac{X_{fh,g}}{X_{fh}}\frac{\partial \ln X_{fh,g}}{\partial t_k}}.$$

The FOC, therefore, can be further simplified as

$$\begin{aligned}
&\frac{\partial V_h}{\partial Y_h}\left\{\tilde{R}^k_{fh}\frac{\partial \ln X_{fh}}{\partial t_k} + \left[Y_h\frac{\partial \ln Y_h}{\partial \ln w} - \sum_g Y_{h,g}\frac{\partial \ln P_{h,g}}{\partial \ln w}\right]\frac{d\ln w}{dt_k}\right\} \\
&\quad = \frac{\partial V_h}{\partial Y_h}\left\{\tilde{R}^k_{fh}\frac{\partial \ln X_{fh}}{\partial t_k} + \left[R_{fh}\frac{\partial \ln R_{fh}}{\partial \ln w} - \lambda_{fh}Y_h\right]\frac{d\ln w}{dt_k}\right\} = 0,
\end{aligned}$$

where the above equation follows from the fact that $\sum_g Y_{h,g}\frac{\partial \ln P_{h,g}}{\partial \ln w} = \sum_g \lambda_{fh,g} Y_{h,g} = \lambda_{fh} Y_h$, and $R_{fh} \equiv \sum_k t_k X_{fh,k}$ and $\tilde{R}^k_{fh} \equiv \tilde{t}_k X_{fh}$. The term in the braces can further simplified as

$$\begin{aligned}
&\tilde{R}^k_{fh}\frac{\partial \ln X_{fh}}{\partial t_k} + \left[\frac{\partial \ln R_{fh}}{\partial \ln w} - \lambda_{fh} Y_h\right]\frac{d\ln w}{dt_k}\\
&= \left\{\left[(1+\epsilon_{hf})\tilde{R}^k_{fh} - \lambda_{fh} Y_h\right] + \left[R_{fh}\frac{\partial \ln R_{fh}}{\partial \ln w} - \tilde{R}^k_{fh}\frac{\partial \ln X_{fh}}{\partial \ln w}\right]\right\}\frac{d\ln w}{dt_k}.
\end{aligned}$$

Therefore, altogether, the FOC becomes

$$\frac{d\ln V_h}{dt_k} = \left\{\left[(1+\epsilon_{hf})\tilde{R}^k_{fh} - \lambda_{fh} Y_h\right] + \left[R_{fh}\frac{\partial \ln R_{fh}}{\partial \ln w} - \tilde{R}^k_{fh}\frac{\partial \ln X_{fh}}{\partial \ln w}\right]\right\}\frac{\partial V_h}{\partial Y_h}\cdot\frac{d\ln w}{dt_k} = 0.$$

Note that $\frac{d\ln w}{dt_k} < 0$ and $\frac{\partial V_h}{\partial Y_h} > 0$, the FOC implies that[28]

$$\begin{aligned}
\frac{d\ln V_h}{dt_k} = 0 \Longleftrightarrow \Gamma(\tilde{t}_k) &\equiv (1+\epsilon_{hf})\tilde{R}^k_{fh} - \lambda_{fh} Y_h + R_{fh}\frac{\partial \ln R_{fh}}{\partial \ln w} - \tilde{R}^k_{fh}\frac{\partial \ln X_{fh}}{\partial \ln w} = 0\\
\Longrightarrow \tilde{t}_k &= \frac{\lambda_{fh} Y_h - R\frac{\partial \ln R_{fh}}{\partial \ln w}}{X_{fh}\left(1+\epsilon_{hf} - \frac{\partial \ln X_{fh}}{\partial \ln w}\right)}\ \forall k.
\end{aligned}$$

Provided that all variables on the right hand side are aggregate variables (i.e., independent of k), the above equation entails that $\tilde{t}^*_1 = ... = \tilde{t}^*_K$. Uniformity of the actual tariffs then simply follows from contradiction. Specifically, suppose $t^*_K = \max\left\{t^*_k\right\}$ and $t^*_1 = \min\left\{t^*_k\right\}$; if $t^*_K \neq t^*_1$ then $\tilde{t}^*_K > \tilde{t}^*_1$, which constitutes a contradiction.[29] Uniformity, in turn, implies that (i) $\tilde{t}^*_k = t^*_k = \bar{t}^*$ for all k, (ii) $\tilde{R}^k_{fh} = R_{fh} = \bar{t}^* X_{fh}$, (iii) $R_{fh}\frac{\partial \ln R_{fh}}{\partial \ln w} = \tilde{R}^k_{fh}\frac{\partial \ln X_{fh}}{\partial \ln w}$, and (vi) $\lambda_{fh} Y_h = (1+\bar{t}^*) X_{fh}$. Plugging these relations into the FOC ($\Gamma(t) = 0$) leads to the following specification for the uniform optimal tariff:

$$\bar{t}^* = \frac{1}{\epsilon_{hf}},$$

where $\epsilon_{hf} \equiv \frac{\partial \ln \lambda_{hf}}{\partial \ln w}$. Given that $\zeta \lambda_{hf} L_f$ is Foreign's aggregate export supply curve (where ζ is a constant), ϵ_{hf} denotes the elasticity of Foreign's aggregate export supply curve. Finally, given that $\frac{\partial V_h}{\partial Y_h}\cdot\frac{d\ln w}{dt_k} < 0$ and that when $\bar{t}^*\epsilon_{hf} < 1$, the expression in the curly

[28]The above expression is the one we highlighted in Section 3 as $\Gamma(t_k, \mathbf{X})$. As we will show next, $\tilde{R}^k_{fh} = t_k X_{hf}$, and the only sector k-specific input into $\Gamma(.)$ is the tariff k.

[29]The fact that $\tilde{t}^*_K > \tilde{t}^*_1$ follows from the fact that $\tilde{t}_k$ is a weighted average tariff with the majority of weight assigned to sector k's tariff—i.e., $\frac{\partial \ln X_{fh}}{\partial t_k} \Longrightarrow |\ \frac{X_{fh,g}}{X_{fh}}\frac{\partial \ln X_{fh,k}}{\partial t_k}\ | > \sum_{g\neq k}\frac{X_{fh,g}}{X_{fh}}\frac{\partial \ln X_{fh,g}}{\partial t_k}$.

bracket is negative, it follows that $\bar{t}^* < \frac{1}{\epsilon_{hf}} \Longleftrightarrow \frac{d\ln V_h}{dt_k} > 0$ and *vice versa.* Hence, it should be the case that the unique solution to the FOC is a global maximum. *Q.E.D.*

Optimal import tariff for an arbitrary set of export policies. Note that $Y_h = L_h + R_{fh} + R^x_{hf}$, where $R_{fh}(w; 1+t_1, ..., 1+t_K) \equiv \sum_{g=1}^K t_g X_{fh,g}$ and $R^x_{hf}(w; 1+x_1, ..., 1+x_K) \equiv \sum_{k=1}^K \frac{x_k}{1+x_k} X_{hf,k}$ denote export and import tax revenue, respectively. Hence, the first order conditions (FOC) under an arbitrary export tax

$$\begin{aligned} \frac{d\ln V_h}{dt_k} &= \frac{\partial V_h}{\partial Y_h} \frac{dY_h}{dt_k} - \sum_g \frac{\partial V_h}{\partial P_{h,g}} \frac{dP_{h,g}}{dt_k} \\ &= \left\{ \frac{\partial R^x_{hf}}{\partial w} \frac{dw}{dt_k} + \frac{d}{dt_k} \left(L_h + R_{fh} \right) \right\} \frac{\partial V_h}{\partial Y_h} - \sum_g \frac{\partial V_h}{\partial P_{h,g}} \frac{dP_{h,g}}{dt_k}. \end{aligned}$$

We can apply a uniform export subsidy $(1+a)$ and a countervailing uniform import tariff $(1+a)$ without altering equilibrium outcomes (this is due to the Lerner symmetry). In that case, at the optimum tariff $(t_1^*, ..., t_K^*)$, the FOC should still hold for any given a:

$$\begin{aligned} \left\{ \frac{\partial R^x_{hf}(w; \frac{1+x_1}{1+a}, ..., \frac{1+x_K}{1+a})}{\partial w} \frac{dw}{dt_k} + \frac{d}{dt_k} R_{fh}\left(w; (1+t_1)(1+a), ..., (1+t_k)(1+a)\right) \right\} \frac{\partial V_h}{\partial Y_h} \\ - \sum_g \frac{\partial V_h}{\partial P_{h,g}} \frac{dP_{h,g}}{dt_k} = 0. \end{aligned}$$

Note that $\Psi(a) \equiv \frac{\partial R^x_{hf}(w; \frac{1+x_1}{1+a}, ..., \frac{1+x_K}{1+a})}{\partial w}$ is continuous in a, $\Psi(0) > 0$ and $\Psi(\max_k\{x_k\}) < 0$. Therefore, following the intermediate value theorem, there exists an $\bar{a}$ such that the $\Psi(\bar{a}) = 0$, which may be used to simplify the FOC as follows:

$$\left(\frac{d}{dt_k} R_{fh}\left(w; (1+t_1)(1+\bar{a}), ..., (1+t_k)(1+\bar{a})\right) \right) \frac{\partial V_h}{\partial Y_h} - \sum_g \frac{\partial V_h}{\partial P_{h,g}} \frac{dP_{h,g}}{dt_k} = 0.$$

As we showed in the first part of the proof, this equation implies $\left(1+t_k^*\right)(1+\bar{a}) = 1 + \bar{t}^* = 1 + \frac{1}{\epsilon_{hf}}$. That is, the optimal import tariff should be uniform for any arbitrary schedule of export tax. *Q.E.D.*

B Proof of Proposition 1 [Optimal Trade Tax Schedule]

From Proposition 1 we know that the optimal import tax should be uniform for an arbitrary vector of export taxes. Therefore, we first characterize the optimal export tax for a zero uniform tariff. Then, using the Lerner symmetry we characterize the optimal tax schedule up to a given uniform tariff.

To this end, we proceed by first deriving $\frac{d\ln Y_h}{d(1+x_k)}$ and $\frac{d\ln P_h}{d(1+x_k)}$. Income of Home is given by (note that $w_h \equiv 1$):

$$Y_h = L_h + \sum_g \frac{x_g}{1+x_g} X_{hf,g},$$

Defining $R^x_{hf} \equiv \sum_k \frac{x_k}{1+x_k} X_{hf,k}$, we will have:

$$\frac{dV_h}{d\left(1+x_k\right)} = \frac{\partial V_h}{\partial Y_h}\frac{\partial Y_h}{\partial\left(1+x_k\right)} + \left[\frac{\partial V_h}{\partial Y_h}\frac{\partial Y_h}{\partial \ln w} + \sum_g \frac{\partial V_h}{\partial P_{h,g}}\frac{\partial P_{h,g}}{\partial \ln w}\right]\frac{d\ln w}{d\left(1+x_k\right)}, \tag{5}$$

where

$$\frac{\partial Y_h}{\partial\left(1+x_k\right)} = \frac{X_{hf,k}}{\left(1+x_k\right)^2} + \frac{x_k}{1+x_k} X_{hf,k}\frac{\partial \ln X_{hf,k}}{\partial\left(1+x_k\right)}.$$

Also, given Roy's identity ($\frac{\partial V_h}{\partial P_{h,g}} = -\frac{Y_{h,g}}{P_{h,g}}\frac{\partial V_h}{\partial Y_h}$) and that $\frac{\partial \ln P_{h,g}}{\partial \ln w} = \lambda_{fh,g}$, the term in the bracket can be simplified as

$$\begin{aligned}\frac{\partial V_h}{\partial Y_h}\frac{\partial Y_h}{\partial \ln w} + \sum_g \frac{\partial V_h}{\partial P_{h,g}}\frac{\partial P_{h,g}}{\partial \ln w} &= \frac{\partial V_h}{\partial Y_h}\frac{\partial Y_h}{\partial \ln w} - \frac{\partial V_h}{\partial Y_h}\sum_g \frac{Y_{h,g}}{P_{h,g}}\frac{\partial P_{h,g}}{\partial \ln w} \\ &= \frac{\partial V_h}{\partial Y_h}\left\{R^x_{hf}\frac{\partial \ln R^x_{hf}}{\partial \ln w} - X_{fh}\right\}.\end{aligned}$$

Defining $\epsilon_{hf,k} \equiv \frac{\partial \ln \lambda_{hf,k}}{\partial \ln w} = \frac{\partial \ln \lambda_{hf,k}}{\partial \ln(1+x_k)}$, and noting that $\alpha_{i,k}$ is constant for the case of Cobb-Douglas utility aggregator, we can write $\frac{\partial \ln X_{hf,k}}{\partial(1+x_k)} = -\epsilon_{hf,k}/\left(1+x_k\right)$ and

$$R^x_{hf}\frac{\partial \ln R^x_{hf}}{\partial \ln w} = X_{hf}\sum \frac{x_g}{1+x_g}\left(1+\epsilon_{hf,g}\right)\frac{X_{hf,g}}{X_{hf}}.$$

Plugging these equations into Equation 5, therefore, leads to the following expression for $\frac{dV_h}{d(1+x_k)}$:

$$\frac{dV_h}{d(1+x_k)} = \frac{\partial V_h}{\partial Y_h}\left\{\frac{1}{(1+x_k)^2}\frac{X_{hf,k}}{X_{hf}}\left[1-x_k\epsilon_{hf,k}\right] + \left\{-1+\sum_g \frac{x_g}{1+x_g}\left(1+\epsilon_{hf,g}\right)\frac{X_{hf,g}}{X_{hf}}\right\}\frac{d\ln w}{d(1+x_k)}\right\}.$$

Hence, given that $\frac{\partial V_h}{\partial Y_h} > 0$, the above equation implies the following FOC for the sector k:

$$\frac{1}{1+x_k}\frac{X_{hf,k}}{X_{hf}}\left[1-x_k\epsilon_{hf,k}\right] + \left\{\sum_g \frac{1}{1+x_g}\frac{X_{hf,g}}{X_{hf}}\left(x_g\epsilon_{hf,g}-1\right)\right\}\frac{d\ln w}{d\ln(1+x_k)} = 0. \tag{6}$$

Balanced trade (R2) entails that $\frac{d\ln w}{d\ln(1+x_k)} = \frac{\frac{\partial \ln X_{hf}}{\partial \ln(1+x_k)}}{\frac{\partial \ln X_{fh}}{\partial \ln w} - \frac{\partial \ln X_{hf}}{\partial \ln w}} = \frac{\frac{X_{hf,k}}{X_{hf}}\frac{\partial \ln X_{hf,k}}{\partial \ln(1+x_k)}}{\frac{\partial \ln X_{fh}}{\partial \ln w} - \frac{\partial \ln X_{hf}}{\partial \ln w}}$. Substituting this in the FOC (6) and noting that $\epsilon_{hf,k} \equiv \frac{\partial \ln X_{hf}}{\partial \ln(1+x_k)}$, yields

$$\frac{1-x_k\epsilon_{hf,k}}{(1+x_k)\epsilon_{hf,k}} = -\frac{\sum_g \frac{1}{1+x_g}\frac{X_{hf,g}}{X_{hf}}\left(x_g\epsilon_{hf,g}-1\right)}{\frac{\partial \ln X_{fh}}{\partial \ln w} - \frac{\partial \ln X_{hf}}{\partial \ln w}}.$$

Since the RHS of this equation is the same for all sectors, we have $\frac{1-x_k\epsilon_{hf,k}}{(1+x_k)\epsilon_{hf,k}} = \frac{1-x_g\epsilon_{hf,g}}{(1+x_g)\epsilon_{hf,g}}$, which is satisfied iff $x_k\epsilon_{hf,k} = x_g\epsilon_{hf,g} \equiv \omega$ for all k and g. Substituting ω in the FOC (6) implies:

$$(1-\omega)\left\{\frac{1}{1+x_k}\frac{X_{hf,k}}{X_{hf}} - \left\{\sum_g \frac{1}{1+x_g}\frac{X_{hf,g}}{X_{hf}}\right\}\frac{d\ln w}{d\ln(1+x_k)}\right\} = 0,$$

which given that $0 < \frac{d\ln w}{d\ln(1+x_k)} < 1$ implies that $\omega = 1$ is the unique solution to the FOC.[30] That is, $x_k^* = 1/\epsilon_{hf,k}^*$ for all k. Given the Lerner Symmetry, for a given uniform tariff t^* the optimal tax schedule is thus uniquely given by

$$(1+\bar{t}^*)(1+x_k^*) = 1 + \frac{1}{\epsilon_{hf,k}^*}.$$

Q.E.D.

[30] Otherwise, if $\frac{1}{1+x_k}\frac{X_{hf,k}}{X_{hf}} - \left\{\sum_g \frac{1}{1+x_g}\frac{X_{hf,g}}{X_{hf}}\right\}\frac{d\ln w}{d\ln x_k} = 0$, $\forall k$ then we would have $\frac{d\ln w}{d\ln x_k} = 1$.

C Special Cases

C.1 Ricardian Model With Many Homogeneous Sectors ($\theta_k \to \infty$)

A special case of our framework is a model in which all sectors are homogeneous (i.e., $\theta_k \to \infty$). This special case is analogous to the Ricardian model of Dornbusch et al. (1977), which has been previously analyzed in Opp (2010) and Costinot et al. (2015). The presence of homogeneous sectors gives rise to knife-edge trade equilibria—where a small shock to the economy could transform an import sector into an export sector. A common strategy to smooth these knife-edge effects involves assuming a continuum of sectors à la Dornbusch et al. (1977). Our framework, however, allows for an alternative smoothing strategy. We retain the discrete structure of the economy, and instead formulate the optimal trade tax problem as the limiting case of the smooth problem characterized by Proposition 1. Hence, Proposition 1, can be used to write the optimal export policy of the homogenous sector k as $\left(1+\bar{t}^*\right)\left(1+x_k^*\right) = 1 + \lim_{\theta_k \to \infty} \frac{1}{\theta_k \lambda_{ff,k}}$. To demonstrate this let us adopt the following decomposition$A_{ji,k} = \left(\tau_{ji,k} a_{j,k}\right)^{\frac{1}{\theta_k}}$, where $a_{j,k}$ denotes actual productivity in sector k and $\tau_{ji,k}$ denotes sector-level iceberg transport costs. We show below that:

$$\lim_{\theta_k \to \infty} \left(1+\bar{t}^*\right)\left(1+x_k^*\right) = \frac{w_f / a_{f,k}}{\tau_{hf,k} w_h / a_{h,k}},$$

The above equation describes the optimal export tax on goods/sectors in which Home has a comparative advantage—i.e., $\frac{\tau_{hf,k} w_h}{a_{h,k}} > \frac{w_f}{a_{f,k}}$. It also provides a closed-form characterization of the *limit-pricing* scheme highlighted in Costinot et al. (2015). Intuitively, the optimality of a limit-pricing tax reflects the Home government's aversion to comparative advantage reversal. Furthermore, the limit-pricing tax entails that the optimal export tax increases with Home's comparative advantage in a given sector.

Overall, depending on the environment, comparative advantage may have a limited role in determining the optimal policy. In particular, the importance of comparative advantage diminishes once we move beyond the borderline case where tariffs can reverse patterns of comparative advantage. In summary, one could argue that across homogeneous sectors/products optimal policy is regulated by the pattern of comparative advantage, whereas as across differentiated sectors/products optimal policy is regulated by sector-level trade elasticities.

Derivation of Limit-Pricing Formula. Given our proof of uniqueness, we proceed to show that the solution to our optimal export tax equation satisfies the following when θ_k

is sufficiently large. That is,

$$\lim_{\theta_k\to\infty} 1+x_k^* = \lim_{\theta_k\to\infty} 1+\frac{1}{\theta_k\lambda_{ii,k}} = \lim_{\theta_k\to\infty}\frac{a_{h,k}w}{a_{f,k}\tau_{hf,k}}\left[1-\frac{\ln\left(\left(\frac{a_{h,k}w}{a_{f,k}\tau_{hf,k}}-1\right)\theta_k\right)}{\theta_k}\right], \tag{7}$$

where, as noted earlier, $a_{j,k}$ denotes actual productivity such that $A_{ji,k}=\left(\tau_{ji,k}a_{j,k}\right)^{\frac{1}{\theta_k}}$, and $w\equiv w_f/w_h$. First, note that as $\theta_k\to\infty$ (since $\lim_{x\to 0}x\ln x=0$) the tax rate implied by the above equation converges to the limit-pricing rate:

$$\lim_{\theta_k\to\infty}(1+x_k^*)=\frac{w_f/a_{f,k}}{\tau_{hf,k}w_h/a_{h,k}}.$$

Hence, it suffices to show that $x_k^*+1=\frac{a_{h,k}w}{a_{f,k}\tau_{hf,k}}\left[1-\frac{1}{\theta_k}\ln\left(\frac{a_{h,k}w}{a_{f,k}\tau_{hf,k}}-1\right)\theta_k\right]$ is a solution to our optimal tax-rate formula ($x_k^*=\frac{1}{\theta_k\lambda_{ff,k}}$) as θ_k becomes sufficiently large. To this end, notice that trade shares in sector/good k are given by

$$\lambda_{ff,k}=\frac{\left(w/a_{f,k}\right)^{-\theta_k}}{\left(w/a_{f,k}\right)^{-\theta_k}+\left(\left[1+x_k\right]\tau_{hf,k}/a_{h,k}\right)}=\frac{\left(\frac{(1+x_k)\tau_{hf,k}a_{f,,k}}{wa_{h,k}}\right)^{\theta_k}}{1+\left(\frac{(1+x_k)\tau_{hf,k}a_{f,,k}}{wa_{h,k}}\right)^{\theta_k}}.$$

Taking the limit of the optimal tax equation yields

$$\lim_{\theta_k\to\infty}\frac{1}{\theta_k\lambda_{ff,k}}=\lim_{\theta_k\to\infty}\frac{1}{\theta_k\left(\frac{(1+x_k)\tau_{hf,k}a_{f,,k}}{wa_{h,k}}\right)^{\theta_k}}.$$

Substituting for x_k from 7 into this equation yields

$$\frac{1}{\lim_{\theta_k\to\infty}\theta_k\left[1-\frac{\ln\theta_k\left(\frac{a_{h,k}w}{a_{f,k}\tau_{hf,k}}-1\right)}{\theta_k}\right]^{\theta_k}}=\frac{1}{1/\left(\frac{a_{h,k}w}{a_{f,k}\tau_{hf,k}}-1\right)}=\lim_{\theta_k\to\infty}x_k^*,$$

which establishes that for sufficiently large θ_k's, the *unique* solution to our optimal tax formula takes the following form, which reflects *limit-pricin*g

$$\lim_{\theta_k \to \infty} 1 + x_k^* = \lim_{\theta_k \to \infty} \frac{a_{h,k}w}{a_{f,k}\tau_{hf,k}} \left[1 - \frac{\ln\left(\left(\frac{a_{h,k}w}{a_{f,k}\tau_{hf,k}} - 1 \right) \theta_k \right)}{\theta_k} \right]$$
$$= \frac{a_{h,k}w}{a_{f,k}\tau_{hf,k}} = \frac{w_f/a_{f,k}}{\tau_{hf,k}w_h/a_{h,k}}.$$

C.2 Cross-Elasticity Effects in the Special case of CES

Consider a gravity model that features CES utility aggregator across sectors. By analyzing this case, we demonstrate how the widely-used Cobb-Douglass utility aggregator is not fully innocuous as far calculating the optimal tariffs go. To be specific, consider the following utility function across sectors:

$$U_i\left(Q_{i,1}, ..., Q_{i,K}\right) = \left[\sum_{k=1}^{K} Q_{i,k}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}.$$

The above utility implies that $\lambda_{hf} = \sum \alpha_{f,k}\lambda_{fh,k}$ where $\alpha_{f,k} \equiv (P_{h.k}/P_h)^{1-\sigma}$—with $P_h = \left(\sum_g P_{h,g}^{1-\sigma} \right)^{1/(1-\sigma)}$ denoting Home's aggregate price index. The CES case, therefore, accommodates cross-substitution effects. Following Proposition 1, the optimal import tariff (conditional on no export policy) remains uniform, but is described by the following equation:

$$\bar{t}^* = \frac{1}{\tilde{\theta}_{hf}\lambda_{ff} + (\sigma - 1)\sum_k \frac{\alpha_{f,k}\lambda_{hf,k}}{\lambda_{hf}} \left[\lambda_{hf,k} - \lambda_{hf} \right]} > \frac{1}{\tilde{\theta}_{hf}\lambda_{ff}},$$

where the inequality follows from the fact that $\sum_k \alpha_{f,k}\lambda_{hf,k} \left[\lambda_{hf,k} - \lambda_{hf} \right] > \sum_k \alpha_{f,k} \left[\lambda_{hf,k} - \lambda_{hf} \right] = 0$. Put simply, the above inequality states that overlooking cross-price elasticity effects (with Cobb-Douglas preferences) may overstate the optimal tariff, especially if the actual cross-sector elasticity, σ, is sizable. Moreover, the greater the degree of Home's sectoral specialization (i.e., the greater the variation in the $\lambda_{hf,k}$'s), the more sensitive the optimal tariff level to σ.

D Proof of Proposition 2 [Tariff Complementarity]

Let R denote the set of liberalized sectors—i.e., $t_k = 0$ if $k \in R$. Building on the the proof of Proposition 1, we first write the FOC condition for the unrestricted sectors. For an unrestricted sector k the FOC implies that $\tilde{t}_k^* = \bar{t}^* = \frac{\lambda_{fh}Y_h - R_{fh}\frac{\partial \ln R_{fh}}{\partial \ln w}}{X_{fh}\left(1+\epsilon_{hf}-\frac{\partial \ln X_{fh}}{\partial \ln w}\right)}$. Given that $\tilde{t}_k^*$ are uniform, the unique solution to the FOC should consists of uniform vector of actual tariffs in the unrestricted sectors: $t_k^* = \tilde{t}_k^* = \bar{t}^*$ for all $k \notin R$. To determine $\bar{t}^*$ we simply need to solve the following:

$$\bar{t}^* = \frac{\lambda_{fh}Y_h - R_{fh}\frac{\partial \ln R_{fh}}{\partial \ln w}}{X_{fh}\left(1+\epsilon_{hf}-\frac{\partial \ln X_{fh}}{\partial \ln w}\right)},$$

where given the uniformity of the actual tariffs in the unrestricted sectors we have (i) $\lambda_{fh}Y_h = X_{fh} + \bar{t}^* \sum_{k \notin R} X_{fh,k}$, and (ii) $R_{fh}\frac{\partial \ln R_{fh}}{\partial \ln w} = \bar{t}^* \sum_{k \notin R} \frac{\partial X_{fh,k}}{\partial w}$. Plugging the former relations in the above equation leads to the following formula for the optimal tariff in unrestricted sectors:

$$t_k^* = \bar{t}^* = \frac{1}{\epsilon_{hf} + \sum_{g \in R} \frac{X_{hf,g}}{X_{hf}}\left(1 - \frac{\partial \ln X_{fh,g}}{\partial \ln w}\right)}, \quad \forall k \notin R,$$

where given that $\frac{\partial \ln X_{fh,g}}{\partial \ln w} < 0$ the above equation implies that $\bar{t}^* < 1/\epsilon_{hf}$, where $1/\epsilon_{hf}$ is the optimal tariff when all sectors set optimal tariffs.

E Optimal Tariff Response Functions

Using the first-order approximation that $\frac{\partial X_{fh}}{\partial t_k} \approx \frac{\partial X_{fh,k}}{\partial t_k}$ (note that as $\frac{X_{fh,k}}{X_{fh}} \to 0$ then $\frac{\partial X_{fh}}{\partial t_k} \to \frac{\partial X_{fh,k}}{\partial t_k}$) we can write the optimal tariff response in sector k as a function applied tariffs in other sectors. In particular, the FOC for sector k can be written as

$$\Gamma_k = (1+\epsilon_{hf})t_k X_{fh} - \lambda_{fh}Y_h + R_{fh}\frac{\partial \ln R_{fh}}{\partial \ln w} - t_k X_{fh}\frac{\partial \ln X_{fh}}{\partial \ln w} = 0.$$

Note that

$$\begin{aligned}\left(1+\epsilon_{hf}\right) t_k X_{fh} - Y_h \lambda_{fh} &= \left(1+\epsilon_{hf}\right) \sum t_k X_{fh,g} - \sum \left(1+t_g\right) X_{fh,g} \\ &= \left(t_k \epsilon_{fh} - 1\right) X_{fh} + \sum_g \left(t_k - t_g\right) X_{fh,g}.\end{aligned}$$

Similarly,

$$R_{fh}\frac{\partial \ln R_{fh}}{\partial \ln w} - \tilde{R}_{fh}^k \frac{\partial \ln X_{fh}}{\partial \ln w} = -\sum_g \left(t_k - t_g\right) X_{fh,g} \frac{\partial \ln X_{fh,g}}{\partial \ln w}.$$

Plugging in the above relations, the FOC becomes

$$\Gamma_k \equiv \left(t_k \epsilon_{hf} - 1\right) X_{fh} + \sum_g \left(t_k - t_g\right) X_{fh,g} \left[1 - \frac{\partial \ln X_{fh,g}}{\partial \ln w}\right] = 0.$$

Based on the above FOC, therefore, sector k's best tariff response will be given by:

$$t_k^*(t_1, ..., t_K) = \frac{1 + \sum_{g \neq k} t_g \frac{X_{fh,g}}{X_{fh}} \left[1 - \frac{\partial \ln X_{fh,g}}{\partial \ln w}\right]}{\epsilon_{hf} + \sum_{g \neq k} \frac{X_{fh,g}}{X_{fh}} \left[1 - \frac{\partial \ln X_{fh,g}}{\partial \ln w}\right]},$$

which indicates that the optimal tariffs in sector k increase with the weighted average of tariffs applied in other sectors.

F Proof of Proposition 4 [Optimality of NRTBs]

Suppose the government disposes of tariff revenues rather than distributing them back to consumers. Noting that $w_h \equiv 1$ and $Y_h = L_h$, the first order conditions (FOC) facing the Home government will be

$$\frac{d \ln V_h}{d \ln \tau_k} = \sum \frac{\partial V_h}{\partial \ln P_{h,g}} \frac{d \ln P_{h,g}}{\partial \tau_g} = -\frac{\partial V_h}{\partial Y_h} \left(Y_{h,k} \frac{\partial \ln P_{h,k}}{\partial \tau_k} + \sum_g Y_{h,g} \frac{\partial \ln P_{h,g}}{\partial \ln w} \frac{d \ln w}{d \tau_k} \right) = 0.$$

Noting that $Y_{h,k} \frac{\partial \ln P_{h,g}}{\partial \ln w} = \tau_g X_{fh,g}$, $Y_{h,k} \frac{\partial \ln P_{h,k}}{\partial \tau_k} = X_{fh,k}$, and $\frac{\partial V_h}{\partial Y_h} > 0$, then

$$\frac{d \ln V_h}{d \ln \tau_k} > 0 \Longleftrightarrow X_{fh,k} + \frac{d \ln w}{d t_k} \sum_g \tau_g X_{fh,g} < 0.$$

The balanced trade condition entails that $\frac{d \ln w}{d \tau_k} = \frac{\frac{\partial \ln X_{hf}}{\partial \tau_k}}{\frac{d \ln X_{hf}}{d \ln w} - \frac{\partial \ln X_{fh}}{\partial \ln w}}$. Plugging $d \ln w / d\tau_k$ into the above expression will yield

$$\frac{d \ln V_h}{d \ln \tau_k} > 0 \iff X_{fh,k} + \frac{\bar{\tau} X_{fh} \frac{\partial \ln X_{fh}}{\partial t_k}}{\frac{d \ln X_{hf}}{d \ln w} - \frac{\partial \ln X_{fh}}{\partial \ln w}} < 0,$$

where $\bar{\tau} = \sum \tau_k \frac{X_{fh,k}}{X_{fh}}$ is the trade-weighted average tariff. Noting that $\frac{\partial \ln X_{fh}}{\partial \tau_k} = \sum_g \frac{\partial \ln X_{fh,g}}{\partial \tau_k} \frac{X_{fh,g}}{X_{fh}}$, and $\frac{\partial \ln X_{fh,k}}{\partial \tau_k} = \frac{-1}{\tau_k} \left[1 + \theta_k X_{hh,k}\right]$ plus $\frac{d \ln X_{hf}}{d \ln w} = 1 + \epsilon_{hf}$ we will have

$$\frac{d \ln V_h}{d \ln \tau_k} > 0 \iff \frac{X_{fh,k}}{\tau_k} \left\{ 1 - \bar{\tau} \frac{1 + \epsilon_{fh,k}}{1 + \epsilon_{hf} + \epsilon_{fh}} \right\} < 0,$$

where $\epsilon_{fh,k} \equiv |\frac{\partial \ln X_{fh,k}}{\partial \ln w}| = \theta_k \lambda_{hh,k} \alpha_{h,k}$ and $\epsilon_{ji} \equiv |\frac{\partial \ln \lambda_{ji}}{\partial \ln w}| = \sum_k \epsilon_{ji,k} \cdot \frac{X_{fh,k}}{X_{fh}}$, with $\sum_k \frac{X_{ji,k}}{X_{ji}} = 1$ by construction. Given that under all circumstances $\bar{\tau} \geq 1$, then $\tau_k^* = 1$ if $1 > \frac{1+\epsilon_{fh,k}}{1+\epsilon_{hf}+\epsilon_{fh}}$. However, if $1 > \frac{1+\epsilon_{fh,k}}{1+\epsilon_{hf}+\epsilon_{fh}}$, and given that $\bar{\tau} \approx 1$ (with exact equality when $\tau_k^* \to \infty$), then $\tau_k^* \to \infty$. Descriptively, the latter condition requires the elasticity of Home's demand for foreign exports in sector k be greater than the sum of Home and Foreign's *aggregate* import demand elasticities.

G Extensions

In this Appendix we demonstrate how our methodology can be used to solve for the optimal policy in environments featuring input-output linkages and monopolistic competition.

G.1 Optimal Tariffs with Input-Output Linkages

Consider an extended version of our baseline economy with input-output linkages–the framework is reminiscent of Caliendo and Parro (2014). Production combines labor and a composite intermediate input in a Cobb-Douglas fashion, with $\beta \in (0, 1]$ denoting the share of labor in production. Furthermore, as is common in the literature, suppose that the intermediate input uses the same aggregator across goods as the final consumption good: $Q_i^{IM} = \Pi_k Q_{i,k}^{\alpha_{i,k}}$. Given the production structure, the CES price index associated with sector k in country i becomes:

$$P_{h,k} \equiv \left(A_{fh,k} \left[\tau_{fh,k} \left(1 + t_k\right) w_f^{\beta} P_f^{1-\beta} \right]^{-\theta_k} + A_{hh,k} \left[w_h^{\beta} P_h^{1-\beta} \right]^{-\theta_k} \right)^{-\frac{1}{\theta_k}},$$

where $P_i \equiv \Pi_k P_{i,k}^{\alpha_{i,k}}$. Finally, Home's total income, which is the sum of wage income and tariff revenues, may be written as $Y_h = \frac{w_h L_h}{\beta} + \sum_k t_k X_{fh,k}$. We can state our main finding as:

Proposition 5. *In the presence of basic input-output linkages, the optimal tariffs are non-uniform across sectors, and they are described by*

$$t_k^* \approx \bar{t} + \frac{1-\beta}{\Psi\beta}\frac{X_{fh,k}}{X_{fh}}\frac{w_h L_h/\beta}{Y_h}\left(\frac{d\ln\tilde{w}}{dt_k}\right)^{-1},$$

where the uniform component of tariffs, $\bar{t}$, and Ψ are composed of, and only of, aggregate variables, plus $\tilde{w} \equiv w_f^{\beta} P_f^{1-\beta}$.

Proof. Define $\tilde{w}_i \equiv w_i^{\beta} P_i^{1-\beta} = w_i \left(\frac{w_i}{P_i}\right)^{-(1-\beta)}$ and normalize $\tilde{w}_h \equiv 1$. Hence, it immediately follows that $d\ln w_i = d\ln\tilde{w}_i + (1-\beta)\, d\ln\frac{w_i}{P_i}$, where $d\ln\frac{w_i}{P_i}$ is given by:[31]

$$\begin{cases} (1-\beta)\, d\ln\frac{w_h}{P_h} = -\frac{1-\beta}{\beta}\left(\sum_g \alpha_g \lambda_{fh,g} d\ln\tilde{w} + \frac{\alpha_k\lambda_{fh,k}}{1+t_k}\right) = -\frac{1-\beta}{\beta}\left(\lambda_{fh} d\ln\tilde{w} + \frac{X_{fh,k}}{Y_h}\right) \\ (1-\beta)\, d\ln\frac{w_f}{P_f} = -\frac{1-\beta}{\beta}\left\{\sum -\alpha_k\lambda_{hf,k}\right\} d\ln\tilde{w} = \frac{1-\beta}{\beta}\lambda_{hf} d\ln\tilde{w} \end{cases}$$

It, therefore, follows that:

$$\begin{cases} \frac{d\ln w_h}{dt_k} = -\frac{1-\beta}{\beta}\left(\lambda_{fh}\frac{d\ln\tilde{w}}{dt_k} + \frac{X_{fh,k}}{Y_h}\right) \\ \frac{\partial\ln w_f}{\partial t_k} = \left(1 + \frac{1-\beta}{\beta}\lambda_{hf}\right)\frac{d\ln\tilde{w}}{dt_k} \end{cases}$$

Similar to the benchmark case, the FOC is given by

$$\frac{d\ln W_h}{dt_k} = \frac{\partial\ln Y_h}{\partial t_k} + \frac{\partial\ln Y_h}{\partial\tilde{w}}\frac{d\ln\tilde{w}}{dt_k} - \frac{\partial\ln P_h}{\partial t_k} - \frac{\partial\ln P_h}{\partial\tilde{w}}\frac{d\ln\tilde{w}}{dt_k} = 0,$$

where the R2 imposes that $\frac{d\ln\tilde{w}}{dt_k} = \frac{\partial\ln X_{fh}}{\partial t_k}\Big/\left(\frac{\partial\ln X_{hf}}{\partial\ln\tilde{w}} - \frac{\partial\ln X_{fh}}{\partial\ln\tilde{w}}\right)$. Plugging the $\frac{d\ln\tilde{w}}{dt_k}$ into the above equation implies the following:

$$\begin{aligned} \frac{d\ln W_h}{dt_k} =& \left\{-\frac{1-\beta}{\beta}X_{fh,k}\frac{w_h L_h/\beta}{Y_h} + X_{fh,k} + \tilde{R}_{fh}^k\frac{\partial\ln X_{fh}}{\partial t_k} - Y_h\frac{\partial\ln P_h}{\partial t_k}\right\} \\ &+ \left[w_h L_h\frac{\partial\ln w_h}{\partial\ln\tilde{w}} + R_{fh}\frac{\partial\ln R_{fh}}{\partial\ln\tilde{w}} - Y_h\frac{\partial\ln P_h}{\partial\ln\tilde{w}}\right]\left(\frac{\frac{\partial\ln X_{fh}}{\partial t_k}}{\left(\frac{\partial\ln w_f}{\partial\ln\tilde{w}} + \epsilon_{hf}\right) - \frac{\partial\ln X_{fh}}{\partial\ln\tilde{w}}}\right) \\ &\left\{\tilde{t}_k X_{fh}\frac{\partial\ln X_{hf}}{\partial t_k} - \frac{1-\beta}{\beta}X_{fh,k}\frac{w_h L_h/\beta}{Y_h}\right\} + \left[w_h L_h\frac{\partial\ln w_h}{\partial\ln\tilde{w}} + R_{fh}\frac{\partial\ln R_{fh}}{\partial\ln\tilde{w}} - Y_h\frac{\partial\ln P_h}{\partial\ln\tilde{w}}\right]\left(\frac{\frac{\partial\ln X_{hf}}{\partial t_k}}{\left(\frac{\partial\ln w_f}{\partial\ln\tilde{w}} + \epsilon_{hf}\right) - \frac{\partial\ln X_{fh}}{\partial\ln w}}\right), \end{aligned}$$

31 $\left(\frac{w_i}{P_{i,k}}\right)^{-\beta\theta_k} = \lambda_{ii,k} \Longrightarrow \frac{w_i}{P_i} = \Pi\lambda_{ii,k}^{-\frac{\alpha_k}{\theta_k\beta}} \Longrightarrow \frac{d}{dt_k}\ln\frac{w_i}{P_i} = \frac{1}{\beta}\left\{\sum -\frac{\alpha_k}{\theta_k}\frac{\partial\ln\lambda_{ii,k}}{\partial\ln\tilde{w}}\frac{d\ln\tilde{w}}{dt_k} - \frac{\alpha_k}{\theta_k}\frac{\partial\ln\lambda_{ii,k}}{\partial\ln t_k}\right\}$

where $\tilde{t}_k \equiv \dfrac{\sum_g t_g \frac{X_{fh,g}}{X_{fh}} \frac{\partial \ln X_{fh,g}}{\partial t_k}}{\sum_g \frac{X_{fh,g}}{X_{fh}} \frac{\partial \ln X_{fh,g}}{\partial t_k}} \approx t_k$. Simplifying the above equation, we can write the FOC as:

$$\frac{d \ln W_h}{dt_k} = \left\{ \Psi \tilde{t}_k X_{fh} - \Gamma \right\} \frac{d \ln \tilde{w}}{dt_k} - \frac{1-\beta}{\beta} \frac{X_{fh,k}}{Y_h} w_h L_h = 0,$$

where $\Gamma \equiv Y_h \lambda_{fh} - w_h L_h \frac{\partial \ln w_h}{\partial \ln \tilde{w}} + R_{fh} \frac{\partial \ln R_{fh}}{\partial \ln \tilde{w}}$, and $\Psi = \frac{\partial \ln w_f}{\partial \ln \tilde{w}} + \epsilon_{hf} - \frac{\partial \ln X_{fh}}{\partial \ln w}$. Noting that both Γ and Ψ depend on aggregate variables, the above FOC implies that

$$\tilde{t}_k^* = \bar{t} + \frac{1-\beta}{\Psi \beta} \frac{X_{fh,k}}{X_{fh}} \frac{w_h L_h / \beta}{Y_h} \left(\frac{d \ln \widetilde{w}}{dt_k} \right)^{-1},$$

where $\bar{t} = \Gamma / \Psi X_{hf}$ and $\tilde{t}_k \approx t_k$ is a first-order approximation of t_k. $\square$

Note that if $\beta = 1$ (our baseline framework, where production does not use intermediate inputs), the optimal tariff is uniform. However, if $1 > \beta > 0$, optimal tariffs are no longer uniform across sectors and vary with comparative advantage (through the term $\frac{X_{fh,k}}{X_{fh}}$) and the scale elasticity (through the term $\frac{d \ln \widetilde{w}}{dt_k}$). These findings are depicted in Figure 6 for a model of the US economy calibrated to factual trade plus revenue shares, trade elasticities and applied tariffs under various values of β.

G.2 Monopolistic Competition

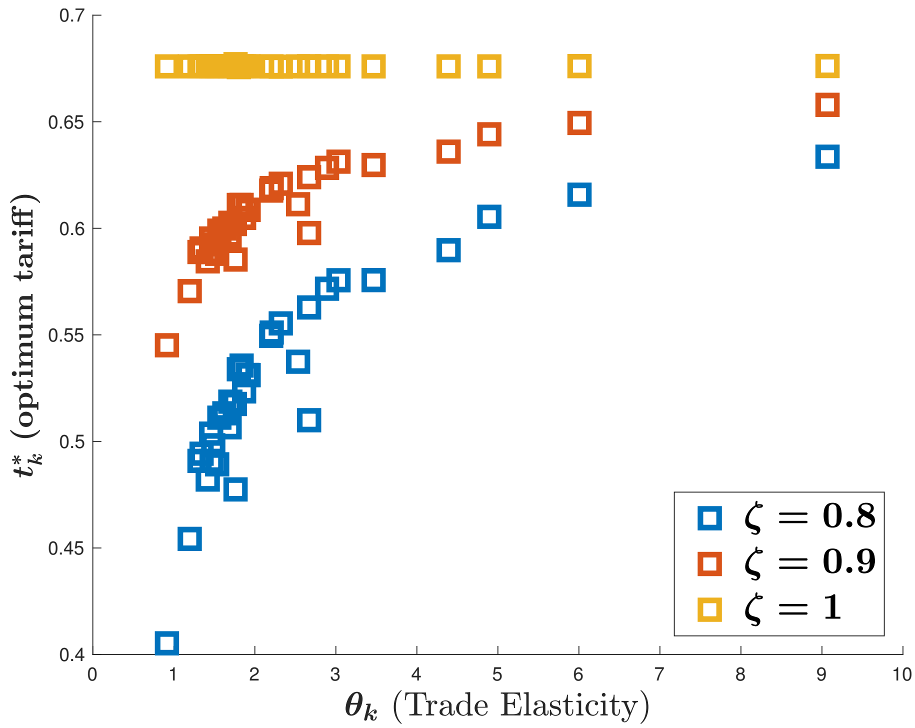
All of our results automatically apply to any monopolistically competitive gravity model that, in addition to satisfying R1-R3, satisfies the following restriction:

R4. *Income from profits are a constant share of Labor Income:* $\frac{\Pi_i}{w_i L_i} = \gamma$.

As shown by Costinot and Rodriguez-Clare (2013), most canonical trade models featuring monopolistic competition satisfy R1.[32] Restriction R3 in a setting with monopolistic competition rules out free entry, thereby creating aggregate profits in the economy. R4 imposes some structure on these aggregate profits. In a multi-sector Krugman model R4 holds when trade elasticities are uniform across sectors (though sectors could remain asymmetric in other aspects such as income-elasticity). R1-R4 also holds in Chaney's (2008) version of the Melitz model where trade and demand elasticities vary across sectors, but aggregate profits are collected by a global fund and redistributed among work-

[32] In monopolistically competitive models with selection (e.g., Chaney (2008)) the trade share equation requires an amendment when taxes are imposed on revenue rather than cost. More specifically, when revenues are taxed, the tariff elasticity is different from the trade elasticity, θ_k.

Figure 6: Optimal tariffs with and without intermediate inputs.



Note: See Section 5 for a description of the data and the calibration strategy.

ers.[33] Overall, all of our results (and corresponding proofs) apply these models because they share the exact same structure as our benchmark model. The only difference (which is irrelevant to our analysis) is that income in country i equals $Y_i = (1+\gamma)\, w_i L_i$, where γ is constant—in our perfectly competitive benchmark model, γ was set to zero.

Overall, R4 eliminates the *profit-shifting* motives behind trade policy, and relaxing it will undermine the uniformity result. However, absent R4, we can still apply our methodology to characterize the optimal tariff structure. To this end, consider a multi-sector Krugman model in the spirit of Ossa (2014). In this model the trade elasticity in sector k corresponds to the elasticity of substitution between firm-specific varieties in that sectors: $\theta_k \sim \sigma_k - 1$. Furthermore, profits in sector k are proportional to total sales: $\pi_{h,k} = \frac{X_{hh,k}+X_{hf,k}}{\sigma_k}$. Provided that $w_i L_i = \sum_k X_{hh,k} + X_{fh,k}$, one can write the share of profit to labor income as:

$$\gamma_i \equiv \frac{\sum \pi_{i,k}}{w_i L_i} = \sum \frac{L_{i,k}}{L_i} \frac{1}{\sigma_k},$$

[33] In addition to the baseline model of Chaney (2008), our theory applies to a general equilibrium variation of the model where all goods are costly to trade and wages are endogenously determined. Regarding R4, Footnote 11 in Chaney (2008) shows that aggregate profits (summed up across countries) are always a constant share, π, of aggregate global labor revenues. Importantly, π depends on only the fundamentals of the global economy and is not affected by trade policy. Chaney (2008) then assumes that profits are collected by a global fund and workers in country i receive $\pi \times w_i L_i$ of the aggregate profits.

where $L_{i,k}$ denotes the number of worker in country i that are employed in sector k, with $L_i = \sum_k L_{i,k}$. Given that $\frac{\partial L_{i,k}}{\partial t_k} > 0$ and $\sum_{g \neq k} \frac{\partial L_{i,g}}{\partial t_k} < 0$, there exists a $\bar{\sigma}_i$ such that if $\sigma_k < \bar{\sigma}_i$ then $\frac{\partial \gamma_i}{\partial t_k} < 0$ and *vice versa*.

Given that $Y_h = (1+\gamma_h) w_h L_h$, the Home government maximizes $V((1+\gamma_h) w_h L_h, \mathbf{P})$. The main distinction between the present problem and our baseline problem is the endogeneity of γ_h to tariffs. In particular, tariffs have profit-shifting effects, such that imposing a tariff on high-profit (low-σ) sectors increases Home's profit income, whereas imposing tariffs on low-profit (high-σ) sectors has the opposite effect. As a result, optimal tariffs are non-uniform and systematically higher in low-σ sectors. These arguments are summarized in the following proposition.

Proposition 6. *In the presence of profit shifting effects (i.e., relaxing R4), optimal tariffs are non-uniform and described by*

$$t_k^* \approx \bar{t} - \frac{\partial \gamma_h}{\partial t_k} \left(\frac{d \ln w}{d t_k} \right)^{-1},$$

where $\bar{t}$ depends on, and only on, aggregate variables. Furthermore, there exists a $\bar{\sigma}$ such that if $\sigma_k < \bar{\sigma}$ ($\sigma_k < \bar{\sigma}$) then $t_k^* > \bar{t}$ ($t_k^* < \bar{t}$).

Proof. First, we write down the first order conditions (FOC), Then, we show that a unique vector of uniform tariffs satisfies the FOC. The FOC corresponding to sector k can be written as:

$$\frac{dV_h}{dt_k} = \frac{\partial V_h}{\partial Y_h}\frac{\partial Y_h}{\partial t_k} + \frac{\partial V_h}{\partial P_{h,k}}\frac{\partial P_{h,k}}{\partial t_k} + \left[\frac{\partial V_h}{\partial Y_h}\frac{\partial Y_h}{\partial \ln w} + \sum_g \frac{\partial V_h}{\partial P_{h,g}}\frac{\partial P_{h,g}}{\partial \ln w} \right] \frac{d \ln w}{d t_k}.$$

Given the balanced trade condition ($X_{fh} = X_{hf}$) the wage effect of a tariff in sector k is given by

$$\frac{d \ln w}{d t_k} = \frac{\frac{\partial \ln X_{fh}}{\partial t_k}}{\frac{\partial \ln X_{hf}}{\partial \ln w} - \frac{\partial \ln X_{fh}}{\partial \ln w}}.$$

Additionally, note that $\frac{\partial \ln X_{hf}}{\partial \ln w} = 1 + \epsilon_{hf}$, where $\epsilon_{hf} \equiv \frac{\partial \ln \lambda_{hf}}{\partial \ln w}$. Roy's identity implies that $\frac{\partial V_h}{\partial P_{h,k}} = -\frac{Y_{h,k}}{P_{h,k}}\frac{\partial V_h}{\partial Y_h}$. Furthermore, $\frac{\partial \ln P_{h,k}}{\partial t_k} = \frac{\lambda_{fh,k}}{1+t_k}$ and $\frac{\partial \ln P_{h,k}}{\partial \ln w} = \lambda_{fh,k}$. Accounting for these relations, the FOC will become

$$\frac{\partial V_h}{\partial Y_h}\left(X_{fh,k} + \tilde{t}_k X_{hf} \frac{\partial \ln X_{fh}}{\partial t_k} + \frac{\partial \gamma_h}{\partial t_k} \right) - \underbrace{\frac{Y_{h,k}}{P_{h,k}}\frac{\partial V_h}{\partial Y_h} P_{h,k} \frac{\lambda_{fh,k}}{1+t_k}}_{X_{fh,k}\frac{\partial V_h}{\partial Y_h}} + \left[\frac{\partial V_h}{\partial Y_h}\frac{\partial Y_h}{\partial w} + \sum_g \frac{\partial V_h}{\partial P_{h,g}}\frac{\partial P_{h,g}}{\partial w} \right] \frac{dw}{dt_k} = 0,$$

where $\tilde{t}_k \equiv \frac{\sum_g t_g \frac{X_{fh,g}}{X_{fh}} \frac{\partial \ln X_{fh,g}}{\partial t_k}}{\sum_g \frac{X_{fh,g}}{X_{fh}} \frac{\partial \ln X_{fh,g}}{\partial t_k}}$ —note that if $\frac{\partial \ln X_{fh,g}}{\partial t_k} \approx 0$ when $t \neq g$, then $\tilde{t}_k \approx t_k$. The FOC, therefore, can be further simplified as:

$$\frac{\partial V_h}{\partial Y_h} \left\{ \tilde{R}^k_{fh} \frac{\partial \ln X_{fh}}{\partial t_k} + \underbrace{\left[Y_h \frac{\partial \ln Y_h}{\partial \ln w} - \sum_g Y_{h,g} \frac{\partial \ln P_{h,g}}{\partial \ln w} \right]}_{-\Psi} \frac{d \ln w}{d t_k} \right\}$$
$$= \frac{\partial V_h}{\partial Y_h} \left\{ \tilde{t}_k \Gamma \frac{d \ln w}{d t_k} - \Psi \frac{d \ln w}{d t_k} + \frac{\partial \gamma_h}{\partial t_k} \right\} = 0.$$

Therefore,

$$\begin{aligned} \tilde{t}_k \Gamma \frac{d \ln w}{d t_k} - \Psi \frac{d \ln w}{d t_k} + \frac{\partial \gamma_h}{\partial t_k} = 0 \\ \Longrightarrow \tilde{t}_k = \bar{t} - \frac{\partial \gamma_h}{\partial t_k} \left(\frac{d \ln w}{d t_k} \right)^{-1} \end{aligned}$$

There exists a $\bar{\sigma}$ such that $\sigma_k > \bar{\sigma}$ then $\frac{\partial \gamma_h}{\partial t_k} < 0$ and $t^*_k > \bar{t}$. Q.E.D. □